Planning Production Line Capacity to Handle Uncertain Demands for a Class of Manufacturing Systems with Multiple Products

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Outline

• Problem Background
• Problem Description
• Problem Formulation
• Problem Analysis and Solution
• Preliminary Results
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Problem Background

• Manufacturing enterprise globalization
  – Global manufacturing network
    • Production lines globally located
    • Multi-products allocated to plants at different locations

• Market globalization
  – Uncertainty
    • Demand
    • Worldwide competition
    • Product price
Problem Background

• Capacity planning
  – Taken before investment
  – Once determined, the capacity could not be changed easily
  – “a firm’s decisions on very large capital investments affect its competitiveness for the next 10 years.”*

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Problem Description

- A manufacturing network
  - Multiple plants and various products
  - Each plant could produce several kinds of products
Problem Description

• Capacity planning
  – To decide the maximal line production rates for each product at each plant
    • The planned maximal line production rates determine the corresponding investments on facilities (hardware)
• How to find the best configuration of the maximal line production rates (capacity configuration)?
Problem Description

• Objective
  – To achieve maximal total profit

• Factors considered
  – Various cost (see next page for detail)
  – Penalty for underproduction (overproduction not allowed)
  – Key point: Production time of a plant shared (discretely divided) among the products produced by the plant
Cost Profile

• Investment cost on production lines
  – Related to the capacity configuration
• Setup cost of production lines
  – Related to the actual line production rates
• Consumption cost of production
• Labor cost (in normal working time and overtime)
Problem Description

• Objective
  – To maximize the total profit
• Given parameters
  – Various cost, penalty, reward coefficients
• Decision variables
  – Network capacity configuration
• Constrains
  – Line production rate constraint
  – Normal working and overtime hours constraint
  – Non-overproduction constraint
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Problem Formulation

- A stochastic programming problem:

\[
\max_{JPH_0 \geq 0} \int (JPH_0) + \mathcal{D}(JPH_0)
\]

where

\[
\mathcal{D}(JPH_0) = E_d \left[ Q(JPH_0, d) \right]
\]

and

\[
Q(JPH_0, d) = \max_x \left\{ g(x) \mid h(x, JPH_0, d) \leq 0, x \in X \right\}
\]
Problem Formulation

• A Stochastic programming problem
  – First stage decision variables: capacity configuration
    • \( JPH_{ij0} \): Maximal line production rate of product \( j \) in plant \( i \)
    • Fitted together to vector \( JPH_0 \)
    • Have to be determined ahead of the investment and the realization of demands.
  – Second stage decision variables: production arrangement
    • \( JPH_{ijt} \): Actual production line rate run for product \( j \) in plant \( i \) in period \( t \).
    • \( y^n_{ijt} \) (\( y^o_{ijt} \), respectively): Normal working (overtime, respectively) hours distributed to product \( j \) in plant \( i \) in period \( t \).
    • Fitted together to vector \( x \).
\[
\text{max} \left\{ \sum_{i \in I, j \in J, t \in T} \left( r_{ij}^n y_{ij}^n JPH_{ij} + r_{ij}^o y_{ij}^o JPH_{ij} \right) - \sum_{i \in I, j \in J, t \in T} c_{ij} JPH_{ij} - \sum_{i \in I, j \in J} s c_{ij} (JPH_{ij})_0 \right\}
\]

\[
g(x) = \sum_{i \in I, j \in J, t \in T} \left( r_{ij}^n y_{ij}^n JPH_{ij} + r_{ij}^o y_{ij}^o JPH_{ij} \right) - \sum_{i \in I, j \in J, t \in T} c_{ij} JPH_{ij} - \sum_{i \in I, j \in J} s c_{ij} (JPH_{ij})_0
\]

\[
f(JPH_0) = \sum_{i \in I, j \in J, t \in T} \left( r_{ij}^n y_{ij}^n JPH_{ij} + r_{ij}^o y_{ij}^o JPH_{ij} \right) - \sum_{i \in I, j \in J, t \in T} c_{ij} JPH_{ij} - \sum_{i \in I, j \in J} s c_{ij} (JPH_{ij})_0
\]

\[
h(x, JPH_0, d) = h(x, JPH_0, d) \leq 0, x \in X
\]
Problem Formulation

\[
\begin{align*}
\max & \quad \sum_{i \in I, j \in J, t \in T} \left( r_{ij}^n y_{ijt}^n JPH_{ijt} + r_{ij}^o y_{ijt}^o JPH_{ijt} \right) - \sum_{i \in I, j \in J, t \in T} c_{ijt} JPH_{ijt} - \sum_{i \in I, j \in J} s c_{ij} (JPH_{ij0}) \\
\text{s.t.} & \quad JPH_{ijt} \leq JPH_{ij0} \quad \forall i \in I, j \in J, t \in T \\
& \quad \sum_{j \in J} y_{ijt}^n \leq HIW^n_{it} \quad \forall i \in I, t \in T \\
& \quad \sum_{j \in J} y_{ijt}^o \leq HIW^o_{it} \quad \forall i \in I, t \in T \\
& \quad \sum_{i \in I} \left( y_{ijt}^n JPH_{ijt} + y_{ijt}^o JPH_{ijt} \right) \leq d_{jt} \quad \forall j \in J, t \in T \\
& \quad JPH_{ijt} \geq 0, y_{ijt}^n, y_{ijt}^o \in \mathbb{Z} \quad \forall i \in I, j \in J, t \in T
\end{align*}
\]

- Actual line production rate
- Maximal line production rate
- Line production rate constraint
- Normal working hours constraint
- Overtime hours constraint
- Non-overproduction constraint
- Uncertain demand (r.v.)
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Problem Analysis

• Two main difficulties
  – The demand uncertainty makes the objective value estimation very hard.
    \[ f(JPH_0) + E_d [Q(JPH_0, d)] \]
  – Even the second stage problem (without uncertainty) is hard to solve due to its complexity:
    \[ Q(JPH_0, d) = \max_x \{ g(x) | h(x, JPH_0, d) \leq 0, x \in X \} \]
Objective Value Estimation

• Objective value has to be estimated based on demand forecasting.
• To obtain an approximately accurate estimation, large amount of demand instances should be randomly generated and calculated with.
The Second Stage Problem

• The second stage problem

\[ Q(JPH_0, d) = \max_x \{ g(x) \mid h(x, JPH_0, d) \leq 0, x \in X \} \]

  – Given \( JPH_0 \) and \( d \)
  – To find the best production arrangement

• Nonlinearity
  – Constraints with product terms

\[
\sum_{i \in I} \left( y^n_{yt} JPH_{yt} + y^o_{yt} JPH_{yt} \right) \leq d_{jt}
\]

  ==> Polynomial programming problem
The Second Stage Problem

- Consider a simple version of the second stage problem:
  - One plant, various products, one period
  - No overtime allowed
- The KNAPSACK problem is polynomially reducible to this problem.

\[
\begin{align*}
\text{max } & \sum_{j \in J} r_j^n y_j^n JPH_j - \sum_{j \in J} c_j JPH_j \\
\text{s.t. } & JPH_j \leq JPH_{j0}, \quad \forall j \in J \\
& \sum_{j \in J} y_j^n \leq HIW^n \\
& y_j^n JPH_j \leq d_j, \quad \forall j \in J \\
& JPH_j \geq 0, y_j^n \in \mathbb{Z}, \quad \forall j \in J
\end{align*}
\]

NP-hard.
Problem Solution

• First consider the second stage problem
  – Polynomial programming problem
  – NP-hard: no efficient exact solution method for large problem

• Two methods of handling this polynomial programming problem
  – Convert to MIP problem$^2$ (F. Glover, E. Woolsey, 1974) and solve with MIP solving tools (e.g. CPLEX)
Two Methods of Handling Polynomial Programming Problem

- **RLT**
  - Key idea:
    - Reformulation-Linearization/convexification + Branch-and-bound
  - May not find the optimal solution within finite time
- **Convert to MIP problem**
  - Could fine optimal solution with MIP solving tools
  - Computing time increases exponentially with the size of the problem.
Convert to MIP Problem

• Key idea:
  – Replace each product term with an additional variable.
  – Introduce an additional constraint for each replacement so that
    • the additional variable equals to the corresponding product term in any case, and thus
    • the two problems before and after the replacement are equivalent.
Convert to MIP Problem

• Conversion rule used in our problem (demonstration):
  – Product term $u \times v$ ($u \in \{0, 1\}$, $0 \leq v \leq 1$) replaced by variable $W$
  – Additional constraints

\[ u + v - W \leq 1 \]
\[ u \geq W \]
\[ v \geq W \]

\[
\begin{array}{ccc}
\text{u} & \text{v} & \text{W} \\
0 & v_0 & 0 \\
1 & v_0 & v_0 \\
\end{array}
\]
Convert to MIP Problem

• Product terms in our problem

\[ y_{ijt}^n JPH_{ijt} \quad y_{ijt}^o JPH_{ijt} \]

where \( 0 \leq JPH_{ijt} \leq JPH_{ij0} \cdot y_{ijt}^n, y_{ijt}^o \in \mathbb{Z} \).

• Transform into terms having the feature of \( u,v \) \((0 \leq u \leq 1, v \in \{0,1\})\) by variable substitution

\[
\begin{align*}
y_{ijt}^n &= 2^{k-1} y_{ijt,k} + 2^{k-2} y_{ijt,k-1} + \cdots + y_{ijt,1} \\
y_{ijt}^o &= 2^{l-1} y_{ijt,l} + 2^{l-2} y_{ijt,l-1} + \cdots + y_{ijt,1} \\
JPH_{ijt} &= JPH_{ij0} z_{ijt}
\end{align*}
\]

(\( k = \min \{ k \in \mathbb{Z} \mid 2^k > HIW_{it}^n \} \))

(\( l = \min \{ l \in \mathbb{Z} \mid 2^l > HIW_{it}^o \} \))
Problem Solution

• Now consider the capacity planning problem
  – Objective value hard to accurately estimate due to
    • Demand uncertainty
    • NP-hardness of the second stage problem
  – Large search space
    • Assume $I$ plants, $J$ products, and $M$ possible chooses of maximal line production rate for production $j$ at plant $i$ (for any $j \in J$ and any $i \in I$), then
    • Number of possible capacity configurations: $M^{I \times J}$
Problem Solution

• So we turn to Ordinal Optimization (OO)* to find good enough solutions.

Strengths of OO:
– Allow a rough performance estimation model
– Guarantee a high probability to find good enough solutions

OO Applied Solution Framework

• Capacity configuration (design) sampling
  – Uniformly and randomly sample $N$ designs
• Performance estimation
  – Using a rough estimation model
  – OPC type and noise level estimated
• Selecting
  – Horse racing selection rule adopted
• Further distinguishing
Performance Estimation

• A rough estimation model
  – Randomly generate one instance of demand (Bass model* used here for forecasting)
  – For each of the sample designs
    • Evaluate the total profit under the demand instance by solving the second stage problem (Conversion to MIP + CPLEX)
    • The performance of the sample are roughly set to be the profit evaluated

• Estimate the Ordered Performance Curve (OPC) type based on the sorted performances of the $N$ designs.

Introduction to OPC

• Ordered Performance Curve (OPC)
  – A plot of the performance values as a function of the order of performance

• Five OPC types (normalized) *

![Graphs of different OPC types](image-url)
Selecting

• Horse racing selection rule
  – Sort the sample designs according to their estimated performances, and
  – Select the top-s designs as the selected set $S$
    • $s$ depends on the specified good enough set $G$, the required alignment level $k$, the OPC type and the noise level.
    • $s$ could be decided according to the Universal Alignment Probability (UAP) table given by OO theory.
Further Distinguishing

• To find the best from the selected designs
  – Generate more instances of demand
  – For each of the top-s designs
    • Evaluate its performances under each of the instances
    • Average the performances to obtain a more accurate performance estimation
  – Select the design with the best average performance as the final solution
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Second Stage Problem Example

• Problem settings
  – 2 plants, 3 products, and 1 period
  – Given capacity configuration $JPH_0$ and demand $d$

$$JPH_0 = \begin{bmatrix} 0 & 50 & 50 \\ 50 & 50 & 0 \end{bmatrix}, \quad d = [5000, 5000, 5000]$$

  – Normal working hours $HIW^n = [120, 120]$
  – Overtime hours $HIW^o = [24, 24]$
  – Other coefficients are set such that

  • the rewards of producing per unit of product 1 and 2 are the same, and are higher than producing per unit of product 3.
Second Stage Problem Example

• Results
  – Actual line production rate
    \[
    \begin{bmatrix}
    JPH_y \\
    \end{bmatrix} =
    \begin{bmatrix}
    0 & 41.67 & 0 \\
    34.72 & 0 & 0
    \end{bmatrix}
    \]
  – Normal working and overtime hours distribution
    \[
    \begin{bmatrix}
    y^w_y \\
    \end{bmatrix} =
    \begin{bmatrix}
    0 & 120 & 0 \\
    120 & 0 & 0
    \end{bmatrix}
    \]
    \[
    \begin{bmatrix}
    y^o_y \\
    \end{bmatrix} =
    \begin{bmatrix}
    0 & 0 & 0 \\
    24 & 0 & 0
    \end{bmatrix}
    \]
Example with OO Applied

• Problem settings
  – 2 plants, 3 products, and 12 period
  – $JPH_{ij0} \in \{0, 10, 20, \ldots, 100\}$, for any $i$ and any $j$
OO Applied Example

• 1000 design samples uniformly sampled and estimated
  - Performance estimating time
    • Rough performance (total profit) estimation for 1 sample design: ≈10s
    • Total time: ≈ 1000*10s ≈ 3h
  - Further distinguishing time (s = 30)
    • Each selected design further estimated with 27 demand instances
    • Total time: ≈ s*27*10s ≈ 2.5h
OO Applied Example

- Normalized OPC
- Noise level $W$
  - Assume worst case

A Bell type OPC.
OO Applied Example

• Select the top 30 designs in the 1000 to insure
  \[ P \left[ \left| G \cap S \right| \geq 1 \right] \geq 0.95 \]
  where G = set of top 5% designs.

• The solution with the best average performance (after further distinguishing)
  \[ JPH_0 = \begin{bmatrix} 0 & 80 & 20 \\ 70 & 10 & 0 \end{bmatrix} \]
Summary

• Capacity planning problem
  – A stochastic programming problem
  – Objective value hard to estimate
  – NP-hardness of second stage problem

• Solution
  – OO applied solution framework
  – Second stage problem converted to MIP

• Preliminary Results