Planning Production Line Capacity to Handle Uncertain Demands for a Class of Manufacturing Systems with Multiple Products

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Outline

- Problem Background
- Problem Description
- Problem Formulation
- Problem Analysis and Solution
- Preliminary Results

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Problem Background

- Manufacturing enterprise globalization
  - Global manufacturing network
    - Production lines globally located
    - Multi-products allocated to plants at different locations
- Market globalization
  - Uncertainty
    - Demand
    - Worldwide competition
    - Product price

Problem Background

- Capacity planning
  - Taken before investment
  - Once determined, the capacity could not be changed easily
  - "a firm's decisions on very large capital investments affect its competitiveness for the next 10 years."*

* B. Fleischmann, S. Ferber and P. Henrich, "*Strategic Planning of BMW's Global Production Network*," Interfaces 36(3): 194-208, 2006.

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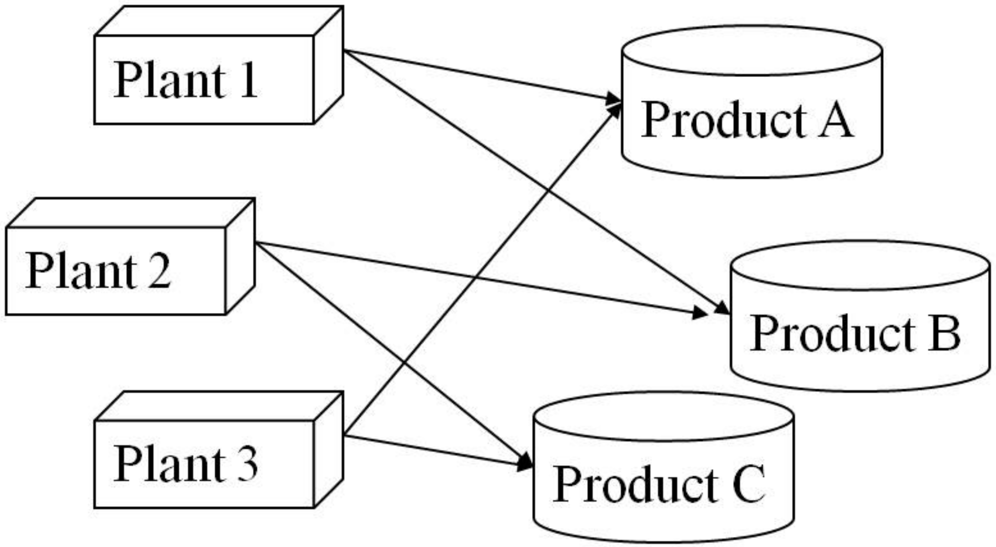
Problem Description

- A manufacturing network
  - Multiple plants and various products
  - Each plant could produce several kinds of products



Problem Description

- Capacity planning
  - To decide the maximal line production rates for each product at each plant
    - The planned maximal line production rates determine the corresponding investments on facilities (hardware)
- How to find the best configuration of the maximal line production rates (capacity configuration)?

Problem Description

- Objective
  - To achieve maximal total profit
- Factors considered
  - Various cost (see next page for detail)
  - Penalty for underproduction (overproduction not allowed)
  - Key point: Production time of a plant shared (discretely divided) among the products produced by the plant

Cost Profile

- Investment cost on production lines
  - Related to the capacity configuration
- Setup cost of production lines
  - Related to the actual line production rates
- Consumption cost of production
- Labor cost (in normal working time and overtime)

Problem Description

- Objective
  - To maximize the total profit
- Given parameters
  - Various cost, penalty, reward coefficients
- Decision variables
  - Network capacity configuration
- Constrains
  - Line production rate constraint
  - Normal working and overtime hours constraint
  - Non-overproduction constraint

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Problem Formulation

- A stochasti r lem:

Network Investment (negative)

Expected Profit

$$\max_{JPH_0 \geq 0} f(JPH_0) + \mathscr{Q}(JPH_0)$$

Network capacity

where

$$\mathscr{Q}(JPH_0) = E_d\left[Q(JPH_0, d)\right]$$

Uncertain deman

and

$$Q(JPH_0, d) = \max_{x} \left\{ g(x) \middle| h(x, JPH_0, d) \leq 0, x \in X \right\}$$

Production arrangement

Problem Formulation

- A Stochastic programming problem
  - First stage decision variables: capacity configuration
    - JPH_{ij0}: Maximal line production rate of product j in plant i
    - Fitted together to vector $\boldsymbol{JPH_0}$
    - Have to be determined ahead of the investment and the realization of demands.
  - Second stage decision variables: production arrangement
    - JPH_{ijt} : Actual production line rate run for product j in plant i in period t.
    - y^n_{ijt} (y^o_{ijt}, respectively): Normal working (overtime, respectively) hours distributed to product j in plant i in period t.
    - Fitted together to vector $\boldsymbol{x}$.

$g(\boldsymbol{x})$ $\quad f(\boldsymbol{JPH_0})$

$$\max \left[\sum_{i\in I, j\in J, t\in T} \left(r^n_{ijt} y^n_{ijt} JPH_{ijt} + r^o_{ijt} y^o_{ijt} JPH_{ijt} \right) - \sum_{i\in I, j\in J, t\in T} c_{ijt} JPH_{ijt} - \sum_{i\in I, j\in J} sc_{ij} (JPH_{ij0}) \right]$$

$$s.t. \begin{cases} JPH_{ijt} \le JPH_{ij0} & \forall i \in I, j \in J, t \in T \\ \sum_{j\in J} y^n_{ijt} \le HIW^n_{it} & \forall i \in I, t \in T \\ \sum_{j\in J} y^o_{ijt} \le HIW^o_{it} & \forall i \in I, t \in T \\ \sum_{i\in I} \left(y^n_{ijt} JPH_{ijt} + y^o_{ijt} JPH_{ijt} \right) \le \boldsymbol{d}_{jt} & \forall j \in J, t \in T \\ JPH_{ijt} \ge 0, y^n_{ijt}, y^o_{ijt} \in \mathbb{Z} & \forall i \in I, j \in J, t \in T \end{cases}$$

$\boldsymbol{h}(\boldsymbol{x}, \boldsymbol{JPH_0}, \boldsymbol{d}) \le 0, \boldsymbol{x} \in \boldsymbol{X}$

Problem Formulation

$$\max\left[\sum_{i\in I,j\in J,t\in T}\left(r_{ijt}^{n}y_{ijt}^{n}JPH_{ijt}+r_{ijt}^{o}y_{ijt}^{o}JPH_{ijt}\right)-\sum_{i\in I,j\in J,t\in T}c_{ijt}JPH_{ijt}-\sum_{i\in I,j\in J}sc_{ij}(JPH_{ij0})\right]$$

Actual line production rate (JPH_{ijt})

Maximal line production rate (JPH_{ij0})

$$s.t.\begin{cases} JPH_{ijt}\le JPH_{ij0} & \forall i\in I,j\in J,t\in T \\ \sum_{j\in J}y_{ijt}^{n}\le HIW_{it}^{n} & \forall i\in I,t\in T \\ \sum_{j\in J}y_{ijt}^{o}\le HIW_{it}^{o} & \forall i\in I,t\in T \\ \sum_{i\in I}\left(y_{ijt}^{n}JPH_{ijt}+y_{ijt}^{o}JPH_{ijt}\right)\le \boldsymbol{d}_{jt} & \forall j\in J,t\in T \\ JPH_{ijt}\ge 0,\ y_{ijt}^{n},y_{ijt}^{o}\in\mathbb{Z} & \forall i\in I,j\in J,t\in T \end{cases}$$

Line production rate constraint (first constraint)

Normal working hours constraint (second constraint)

Overtime hours constraint (third constraint)

Non-overproduction constraint (fourth constraint)

Uncertain demand (r.v.) (d_{jt})

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Problem Analysis

- Two main difficulties
  - The demand uncertainty makes the objective value estimation very hard.

$$f(\boldsymbol{JPH}_0) + E_d\left[Q(\boldsymbol{JPH}_0, \boldsymbol{d})\right]$$

  - Even the second stage problem (without uncertainty) is hard to solve due to its complexity:

$$Q(\boldsymbol{JPH}_0, \boldsymbol{d}) = \max_{\boldsymbol{x}} \left\{ g(\boldsymbol{x}) \middle| \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{JPH}_0, \boldsymbol{d}) \le \boldsymbol{0}, \boldsymbol{x} \in \boldsymbol{X} \right\}$$

Objective Value Estimation

- Objective value has to be estimated based on demand forecasting.
- To obtain an approximately accurate estimation, large amount of demand instances should be randomly generated and calculated with.

The Second Stage Problem

- The second stage problem

$$Q(\boldsymbol{JPH}_0, \boldsymbol{d}) = \max_{\boldsymbol{x}} \left\{ g(\boldsymbol{x}) \middle| \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{JPH}_0, \boldsymbol{d}) \leq \boldsymbol{0}, \boldsymbol{x} \in X \right\}$$

  - Given $\boldsymbol{JPH_0}$ and $\boldsymbol{d}$
  - To find the best production arrangement
- Nonlinearity
  - Constraints with product terms

$$\sum_{i \in I} \left(y^{n}_{ijt} JPH_{ijt} + y^{o}_{ijt} JPH_{ijt} \right) \leq \boldsymbol{d}_{jt}$$

==> Polynomial programming problem

The Second Stage Problem

- Consider a simple version of the second stage problem:
  - One plant, various products, one period
  - No overtime allowed
- The KNAPSACK problem is polynomially reducible to this problem.

NP-hard.

$$\max \sum_{j \in J} r_j^n y_j^n JPH_j - \sum_{j \in J} c_j JPH_j$$

$$s.t. \begin{cases} JPH_j \le JPH_{j0} & \forall j \in J \\ \sum_{j \in J} y_j^n \le HIW^n & \\ y_j^n JPH_j \le d_j & \forall j \in J \\ JPH_j \ge 0, y_j^n \in \mathbb{Z} & \forall j \in J \end{cases}$$

Problem Solution

- First consider the second stage problem
  - Polynomial programming problem
  - NP-hard: no efficient exact solution method for large problem
- Two methods of handling this polynomial programming problem
  - Reformulation-Linearization/convexification Technique (RLT)[1] (H.D. Sherali, C.H. Tuncbilek, 1992)
  - Convert to MIP problem[2] (F. Glover, E. Woolsey, 1974) and solve with MIP solving tools (e.g. CPLEX)

Two Methods of Handling Polynomial Programming Problem

- RLT
  - Key idea:
    - Reformulation-Linearization/convexification + Branch-and-bound
  - May not find the optimal solution within finite time
- Convert to MIP problem
  - Could fine optimal solution with MIP solving tools
  - Computing time increases exponentially with the size of the problem.

Convert to MIP Problem

- Key idea:
  - Replace each product term with an additional variable.
  - Introduce an additional constraint for each replacement so that
    - the additional variable equals to the corresponding product term in any case, and thus
    - the two problems before and after the replacement are equivalent.

Convert to MIP Problem

- Conversion rule used in our problem (demonstration):
  - Product term $u*v$ ($u \in \{0, 1\}$, $0 \le v \le 1$) replaced by variable W
  - Additional constraints

$$u + v - W \le 1$$

$$u \ge W$$

$$v \ge W$$

u	***v***	***W***
0	v_0	0
1	v_0	v_0

Convert to MIP Problem

- Product terms in our problem

$$y_{ijt}^{n} JPH_{ijt} \quad y_{ijt}^{o} JPH_{ijt}$$

where $0 \le JPH_{ijt} \le JPH_{ij0}, y_{ijt}^{n}, y_{ijt}^{o} \in \mathbb{Z}$.

- Transform into terms having the feature of *u*v* (0 ≤ *u* ≤ 1, *v*∈{0, 1}) by variable substitution

$$y_{ijt}^{n} = 2^{k-1} y_{ijt,k}^{n} + 2^{k-2} y_{ijt,k-1}^{n} + \cdots y_{ijt,1}^{n} \quad \left(k = \min\left\{k \in \mathbb{Z} \middle| 2^{k} > HIW_{it}^{n}\right\}\right.$$

$$y_{ijt}^{o} = 2^{l-1} y_{ijt,l}^{o} + 2^{l-2} y_{ijt,l-1}^{o} + \cdots y_{ijt,1}^{o} \quad \left(l = \min\left\{l \in \mathbb{Z} \middle| 2^{l} > HIW_{it}^{o}\right\}\right.$$

$$JPH_{ijt} = JPH_{ij0} z_{ijt}$$

Problem Solution

- Now consider the capacity planning problem
  - Objective value hard to accurately estimate due to
    - Demand uncertainty
    - NP-hardness of the second stage problem
  - Large search space
    - Assume *I* plants, *J* products, and *M* possible chooses of maximal line production rate for production *j* at plant *i* (for any $j \in J$ and any $i \in I$), then
    - Number of possible capacity configurations: M^(I*J)

Problem Solution

- So we turn to Ordinal Optimization (OO)* to find good enough solutions.

  Strengths of OO:

  – Allow a rough performance estimation model

  – Guarantee a high probability to find good enough solutions

* Yu-Chi Ho, Qian-Chuan Zhao, Qing-Shan Jia, *"Ordinal optimization: soft optimization for hard problems,"* Springer, 2007

OO Applied Solution Framework

- Capacity configuration (design) sampling
  - Uniformly and randomly sample *N* designs
- Performance estimation
  - Using a rough estimation model
  - OPC type and noise level estimated
- Selecting
  - Horse racing selection rule adopted
- Further distinguishing

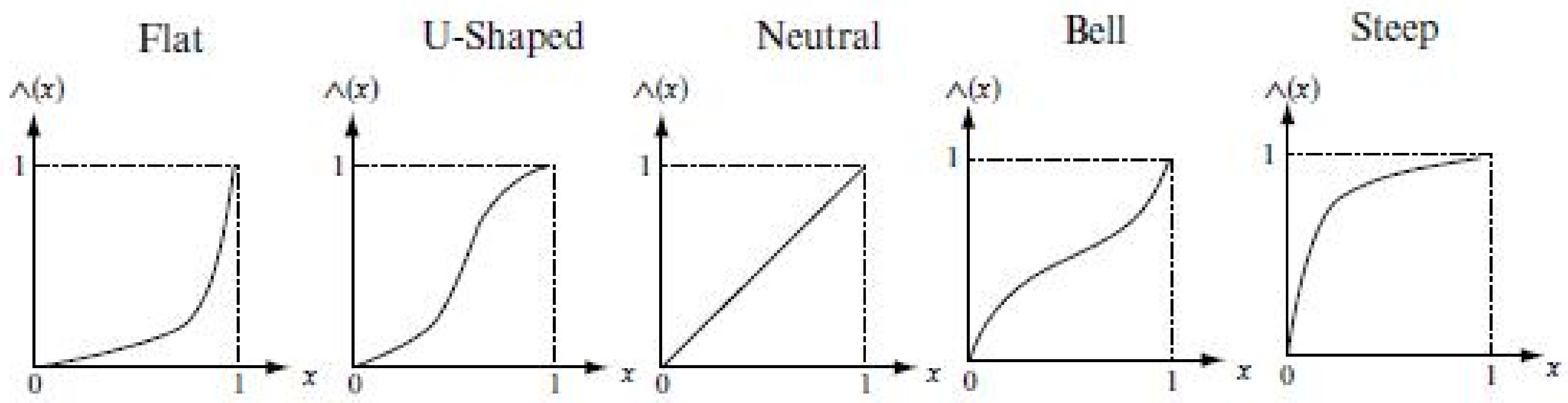
Performance Estimation

- A rough estimation model
  - Randomly generate one instance of demand (Bass model* used here for forecasting)
  - For each of the sample designs
    - Evaluate the total profit under the demand instance by solving the second stage problem (Conversion to MIP + CPLEX)
    - The performance of the sample are roughly set to be the profit evaluated
- Estimate the Ordered Performance Curve (OPC) type based on the sorted performances of the *N* designs.

* F.M. Bass, "*A new product growth model for consumer durables,*" *Management* Science 15:215-227, 1969.

Introduction to OPC

- Ordered Performance Curve (OPC)
  - A plot of the performance values as a function of the order of performance
- Five OPC types (normalized)*



Selecting

- Horse racing selection rule
  - Sort the sample designs according to their estimated performances, and
  - Select the top-*s* designs as the selected set *S*
    - *s* depends on the specified good enough set G, the required alignment level *k*, the OPC type and the noise level.
    - *s* could be decided according to the Universal Alignment Probability (UAP) table given by OO theory.

Further Distinguishing

- To find the best from the selected designs
  - Generate more instances of demand
  - For each of the top-*s* designs
    - Evaluate its performances under each of the instances
    - Average the performances to obtain a more accurate performance estimation
  - Select the design with the best average performance as the final solution

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Second Stage Problem Example

- Problem settings
  - 2 plants, 3 products, and 1 period
  - Given capacity configuration $\boldsymbol{JPH_0}$ and demand $\boldsymbol{d}$

$$\boldsymbol{JPH_0} = \begin{bmatrix} 0 & 50 & 50 \\ 50 & 50 & 0 \end{bmatrix} \qquad \boldsymbol{d} = [5000 \;\; 5000 \;\; 5000]$$

  - Normal working hours $\boldsymbol{HIW}^{n} = [120 \quad 120]$
  - Overtime hours $\boldsymbol{HIW}^{o} = [24 \quad 24]$
  - Other coefficients are set such that
    - the rewards of producing per unit of product 1 and 2 are the same, and are higher than producing per unit of product 3.

Second Stage Problem Example

- Results
  - Actual line production rate

$$\left(JPH_{ij}\right)=\begin{bmatrix} 0 & 41.67 & 0 \\ 34.72 & 0 & 0 \end{bmatrix}$$

  - Normal working and overtime hours distribution

$$\left(y_{ij}^{n}\right)=\begin{bmatrix} 0 & 120 & 0 \\ 120 & 0 & 0 \end{bmatrix} \qquad \left(y_{ij}^{o}\right)=\begin{bmatrix} 0 & 0 & 0 \\ 24 & 0 & 0 \end{bmatrix}$$

Example with OO Applied

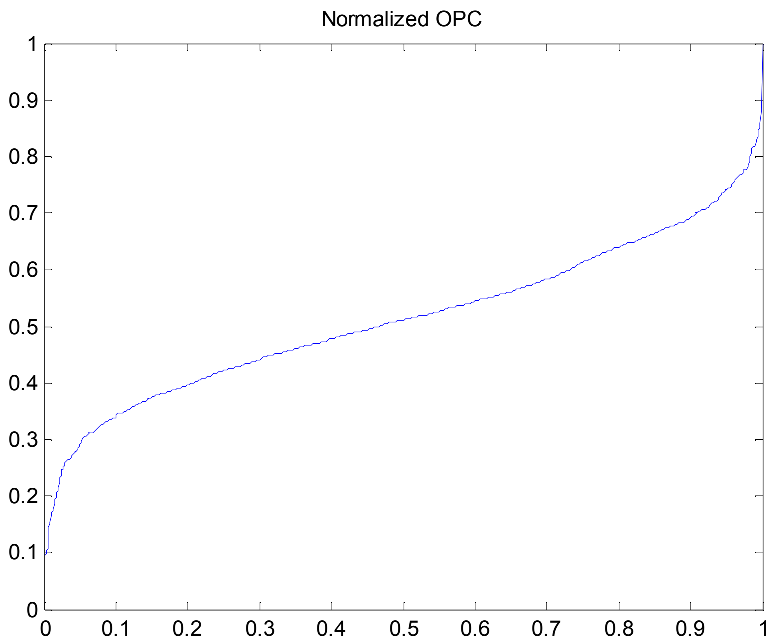
- Problem settings
  - 2 plants, 3 products, and 12 period
  - $JPH_{ij0} \in \{0, 10, 20, \ldots, 100\}$, for any *i* and any *j*

OO Applied Example

- 1000 design samples uniformly sampled and estimated
  - Performance estimating time
    - Rough performance (total profit) estimation for 1 sample design: ≈10s
    - Total time: ≈ 1000*10s ≈ 3h
  - Further distinguishing time (s = 30)
    - Each selected design further estimated with 27 demand instances
    - Total time: ≈ s*27*10s ≈ 2.5h

OO Applied Example

- Normalized OPC



A Bell type OPC.

- Noise level *W*
  - Assume worst case

OO Applied Example

- Select the top 30 designs in the 1000 to insure

$$P\left[\left|G \cap S\right| \geq 1\right] \geq 0.95$$

where G = set of top 5% designs.

- The solution with the best average performance (after further distinguishing)

$$\boldsymbol{JPH}_0 = \begin{bmatrix} 0 & 80 & 20 \\ 70 & 10 & 0 \end{bmatrix}$$

Summary

- Capacity planning problem
  - A stochastic programming problem
  - Objective value hard to estimate
  - NP-hardness of second stage problem
- Solution
  - OO applied solution framework
  - Second stage problem converted to MIP
- Preliminary Results