# Localization of Unknown Networked Radio Sources Using a Mobile Robot with a Directional Antenna 

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#### Abstract

In this paper, we present algorithmic developments for a single mobile robot equipped with a directional antenna to localize unknown networked radio sources. We assume that the robot only senses radio transmissions at the physical layer and that the network has a carrier sense multiple access (CSMA)-type medium access control (MAC) layer. The total number of radio sources are assumed unknown. We first formulate the localization problem and then propose a particle filter-based localization algorithm. The algorithm combines a new CSMA model and a new directional antenna model. The new CSMA model provides network configuration data, such as the network size and the probability of collision, when listening to network traffic. The directional antenna model enhances the efficiency of robot motion. The new combined sensing model is capable of handling transmission collisions during localization. The overall algorithm runs in $O(n m)$ time for $n$ particles and $m$ radio sources at each step. The numerical results show that the algorithm can localize unknown networked radio sources effectively.


## I. Introduction

The recent fast-evolving technology of radio frequency (RF) wireless sensor networks is a double-edged sword. On the one hand, it improves our capability of acquiring information from a remote environment; on the other hand, it provides new espionage tools that threaten our security and privacy. A sensor network is usually composed of a large number of miniature wireless sensor nodes with selfconfigurable ad hoc networking capabilities, which make it very difficult to be localized. When a mobile robot does not understand the communication packet and does not know the total number of radio sources, it is challenging for the robot to perform the localization process because: (1) the periods of radio transmission are short and radio sources may not be active at all times and awaken intermittently and (2) there could be more than one radio source transmitting signals.

We are interested in developing algorithms and schemes to guide robots that are equipped with directional radio antennas to localize unknown networked radio sources. This paper reports the first step of the study in which only one robot is used for the localization task (Fig. 1). Using the equipped directional antenna, the robot can detect variations of radio signal strengths as it travels in the field of radio sources. Although the antenna cannot decode intercepted packets and identify the transmitter by on-the-fly packet content, by listening to the transmission patterns, we can

[^0]still obtain the physical layer information and limited MAC layer information of the wireless network.


Fig. 1. (a) Schematics of deploying a single mobile robot to localize unknown networked radio sources. The radio sources with dashed circles indicate that they are sending radio signals. (b) A prototype of a mobile robot and a radio source.

The recent development of radio frequency-based localization can be viewed as the localization of "friendly" radio sources because researchers assume either a continuously transmitting radio source (similar to a lighthouse) [1]-[4] or known packet information of the network [5]-[9]. Letchner et al. [2] use a network of wireless access points to localize a mobile agent, which can be viewed as a dual version of our problem. They use multiple static listeners to localize a mobile transmitter, while we try to localize multiple static transmitters using a mobile listener. The problem we are facing is more challenging because the robot/listener are not assumed to understand the packet. In [8], Sichitiu and Ramadurai try to localize sensor network nodes with a mobile beacon. However, the scheme requires the communication between the mobile beacon and sensor nodes to identify the transmitter of the radio signal.

In robotics research, simultaneous localization and mapping (SLAM) is the process of mapping the environment and localizing the robot's position at the same time [10][12]. Directly applying SLAM methods to our problem is not appropriate because networked radio sources create a highly dynamic environment where the signal transmission patterns change quickly. A directional log-periodic dipole array (LPDA) antenna is used in our setup because it has excellent wide-band reception. Although physical models of the LPDA antenna have been discussed for more than 20 years [17], unfortunately, for a given antenna, the model cannot provide an accurate prediction of the real signal field due to many random factors, such as the antenna surface and shape. Moreover, the physical model is computationally expensive and cannot be used for real-time applications. Most existing research often simplifies the signal field as an inverse quadratic function with a circular radiation pattern, and such
a simplification cannot capture the antenna's directivity. In this paper, we construct a tractable computational model based on a half wave dipole antenna model.

An early work in [3] uses multiple orthogonal antennas to triangulate the position of a radio source. Although only the physical layer information is assumed for the receiver, the antenna model [3] does not address the problem of multiple simultaneous transmissions, namely, modeling of network traffic. Since a large class of wireless networks use a CSMAbased MAC layer protocol [18], we assume such a protocol as prior knowledge of the network. Incorporating this knowledge into the localization mechanism greatly enhances the system's performance. Existing MAC layer models [19] focus on modeling of MAC protocol behaviors, such as transmission delay and channel capacity and do not provide information for the localization purpose.

Our approach builds on an augmented particle filter that combines three new developments: an antenna sensing model, a probabilistic sensing model, and a CSMA protocol model. The new antenna model characterizes the properties of an LPDA antenna, and the combined probabilistic sensing model predicts sensor readings when there are collisions between multiple transmissions. Combining the information from the three developments, we generate robot control commands to search for and localize radio sources. The overall algorithm runs in $O(\mathrm{~nm})$ time for $n$ particles and $m$ radio sources at each step. Our preliminary simulation results show that the scheme can effectively localize networked radio sources.

The remainder of the paper is organized as follows. We formulate the localization problem in Section II. We also present the system architecture in this section. In Section III, we introduce our sensing model based on the antenna and the CSMA protocol models. The overall algorithm and simulation experiments are presented in Section IV. We conclude the paper with Section V.

## II. Problem Statement and System Design

## A. Problem Statement

We denote the location of the $i$ th radio source as $\left(x_{i}, y_{i}\right)$, $i=1, \ldots, m$, where $m$ is the total number of the radio sources. Denote $\mathbf{x}_{k}=\left\{\left(x_{i}, y_{i}\right) \mid i=1, \ldots, m\right\}$ as the state variable and $\mathbf{u}_{k}$ as the robot motion command at time $k$. We measure the radio signal strength $Z^{k}$ at time $k$, and $\phi_{i j}$ is the carrier phase difference between radio sources $i$ and $j$.

We consider following assumptions.

1. The robot and radio sources are in a free 2D space, and the radio sources are static nodes,
2. The network traffic of the radio sources is light and each transmission is short. Therefore, radio sources cannot be treated as continuous beacons,
3. The radio sources communicate with each other using a CSMA-type MAC protocol,
4. The directional antenna on the robot has high sensitivity and can listen to all traffic,
5. The radiation pattern of the radio sources is circular and the phase difference between any two carrier waves
from radio sources $i$ and $j$ is uniformly, identically, and independently distributed, $\phi_{i j} \sim U[0,2 \pi], i, j=$ $1, \ldots, m$,
6. The robot accurately executes its motion command and we do not consider the mobile robot's dynamics.
We define the localization problem as
Localization Problem: Given the received RF signal strengths, $Z^{1}, Z^{2}, \ldots, Z^{k}$, and robot motions, $\mathbf{u}_{1}, \mathbf{u}_{2}, \ldots, \mathbf{u}_{k}$ at time $k$, identify the number of radio sources $m$ and localize the position of each radio source, $\left\{\left(x_{i}, y_{i}\right)\right\}, i=1,2, \ldots, m$.

## B. System Design

Fig. 2 illustrates the system architecture. The input is the signal strength from the directional LPDA and the output of the system is robot motion commands. The antenna model outputs a probabilistic distribution of transmitter locations according to the LPDA properties and RF wave superposition. The CSMA model listens to the communication traffic and estimates the total number of transmitters. Both the antenna and CSMA models are integrated together in our sensing model. A particle filter is then employed to estimate the posterior probabilistic distribution of states and output such a distribution to the motion generation module. The motion generation module selects the motion that leads the robot toward the most probable radio source location. The robot makes the movement, and the system advances to the next step.


Fig. 2. System architecture.
The localization framework builds around a particle filter. The estimate of the probabilistic distribution $p\left(\mathbf{x}_{k} \mid Z^{k}\right)$ characterizes possible states given current sensor readings. Using Bayes' theorem, $p\left(\mathbf{x}_{k} \mid Z^{k}\right)$ can be computed iteratively as

$$
\begin{align*}
p\left(\mathbf{x}_{k} \mid Z^{k}\right) & =\eta p\left(Z^{k} \mid \mathbf{x}_{k}\right) \\
& \int p\left(\mathbf{x}_{k} \mid \mathbf{x}_{k-1}, \mathbf{u}_{k-1}\right) p\left(\mathbf{x}_{k-1} \mid Z^{k-1}\right) d \mathbf{x}_{k-1} \tag{1}
\end{align*}
$$

where $\eta$ is a normalization constant and $p\left(Z^{k} \mid \mathbf{x}_{k}\right)$ is the measurement model that characterizes the conditional probability distribution of sensor readings $Z^{k}$ given state $\mathbf{x}_{k}$. $p\left(\mathbf{x}_{k} \mid \mathbf{x}_{k-1}, \mathbf{u}_{k-1}\right)$ is the motion model that characterizes the distribution of states $\mathbf{x}_{k}$ given its previous state $\mathbf{x}_{k-1}$ and motion commands $\mathbf{u}_{k-1}$. Computation in Eq. (1) is nontrivial because $p\left(\mathbf{x}_{k} \mid Z^{k}\right)$ is often non-Gaussian and nonlinear. We adopt a Monte Carlo localization method [14] with a sampling/importance sampling (SIS) particle filter.

The localization problem is considered as the following two subproblems. The first subproblem is to compute the conditional probability of the signal strength reading given
known sensor locations $p\left(Z^{k} \mid \mathbf{x}_{k}\right)$. The second subproblem is to estimate the total number of radio sources $m$ based on CSMA modeling. The resulting $m$ and $p\left(Z^{k} \mid \mathbf{x}_{k}\right)$ is fed into Eq. (1). Then, the SIS particle filter is used to compute $p\left(\mathbf{x}_{k} \mid Z^{k}\right)$. Given the current posterior probability $p\left(\mathbf{x}_{k} \mid Z^{k}\right)$ and $m$, the robot motion $\mathbf{u}_{k+1}$ is computed toward the most probable radio source locations. If the robot moves close to a radio source within a preset radius, we consider the radio source to be localized.

## III. System Modeling

The sensing problem is to compute $p\left(Z^{k} \mid \mathbf{x}_{k}\right)$, the probability distribution of the measured signal strength given a known robot and radio source configuration. $Z^{k}$ is a random variable because (1) a radio transmission pattern is a random event and (2) the carrier wave phase difference between collided transmissions is random variable. To compute $Z^{k}$, we need to address the following three problems: (1) What is the antenna reading for an individual transmitter? (2) What is the antenna reading if there is more than one simultaneous transmitter (or a collision)? (3) What are all possible collision types? In the following, we address these three problems sequentially.

## A. Antenna model

1) Single radio source: We denote $r_{i}$ as the distance between the robot and the $i$ th radio source, $\theta_{i}$ as the antenna orientation with respect to the $i$ th radio source, $z_{i}$ as the measured signal strength from the $i$ th radio source, $Z_{\{v\}}$ (a random variable) as the measured signal strength when $v$ radio sources are transmitting signals simultaneously, $Z_{i j}$ (a random variable) as the measured signal strength when both $i$ th and $j$ th radio sources transmit signals simultaneously, and $P_{\text {idle }}$ as the channel idle probability.


Fig. 3. WiNRADiO AX-31B planar log-periodic directional antenna properties.

We use a WiNRADiO AX-31B Planar LPDA in our system (Fig. 3). Because the reception and radiation patterns of an antenna are identical [20], either pattern may be referred to as the "radiation pattern." In order to compute the measured signal strength $z_{i}$, we need to model the LPDA directional antenna. The received signal strength of the directional antenna is given as

$$
\begin{equation*}
z_{i}=C r_{i}^{-\beta} f\left(\theta_{i}\right) \tag{2}
\end{equation*}
$$

where $C$ is a constant depending on transmission power and $r_{i}^{-\beta}$ is the signal decay function. We use the well-accepted


Fig. 4. LPDA antenna gain curve fitting.
decay factor $\beta=2$ [21]. The directivity of the antenna is captured by the term $f\left(\theta_{i}\right)$, which describes the radiation pattern of the antenna.

To obtain an efficient computational model, we use a curve-fitting method based on numerical data from the antenna simulator SuperNEC [22]. We use our LPDA configurations, such as beam lengths and spacing, and the simulator generates the radiation pattern as shown in Fig. 3(b). Note that an LPDA is a wide band antenna with multiple beams and only the beam with length close to half of the wave length is the most effective receiver. Therefore, we use a half wave dipole antenna model to fit the radiation pattern of the LPDA antenna at 433 MHz . We choose this frequency because it is the working frequency of the Berkeley mica2 [23] motes that are used in the experiment.

The radiation pattern in Fig. 3(b) is given by a unit of $d B i$, which is the antenna gain over a theoretical isotropic (point source) antenna. We convert the radiation pattern to an absolute gain as shown by the solid-line curve in Fig. 4. The resulting function is given by
$f\left(\theta_{i}\right)= \begin{cases}1.4825 \cos \theta_{i}, & 0 \leq \theta_{i}<\frac{\pi}{2} \text { or } \frac{3 \pi}{2} \leq \theta_{i}<2 \pi \\ -1.0654 \cos \theta_{i}, & \frac{\pi}{2} \leq \theta_{i}<\frac{3 \pi}{2} .\end{cases}$
The model fits well with simulation data as illustrated in Fig. 4. We then compute the probability distribution of $Z^{k}$ given that only radio source $i$ is transmitting,

$$
p\left(Z^{k} \mid i, \mathbf{x}_{k}\right)=\left\{\begin{array}{lr}
1, & Z^{k}=\left\lfloor z_{i}+0.5\right\rfloor  \tag{4}\\
0, & \text { otherwise }
\end{array}\right.
$$

where $\rfloor$ is the floor function. The model in Eq. (4) considers that the antenna only takes integer readings and there is no randomness in a single radio source.
2) Two or more radio sources: Assume that there are $v, 1<v \leq m$, radio sources transmitting at the same time. Each radio source $i$ has amplitude $A_{i}$ and phase $\phi_{i}$, $i=1,2, \ldots, v$. The wave $\Psi_{i}$ caused by radio source $i$ can be described with a wave function, $\Psi_{i}=A_{i} \cos \left(\omega t+\phi_{i}\right)$, where $\omega$ is the carrier frequency. According to the theory of wave superposition, the combined wave $\Psi=$ $\sum_{i=1}^{v} A_{i} \cos \left(\omega t+\phi_{i}\right)=A \cos (\omega t+\phi)$, where $A$ is the combined amplitude with

$$
\begin{equation*}
A^{2}=\sum_{i=1}^{v} A_{i}^{2}+2 \sum_{i=1}^{v} \sum_{j>i}^{v} A_{i} A_{j} \cos \left(\phi_{i}-\phi_{j}\right) \tag{5}
\end{equation*}
$$

Based on RF theory, the received signal strength is proportional to the square of the wave amplitude, i.e. $z_{i} \sim A_{i}^{2}$. Therefore, we have

$$
\begin{equation*}
Z_{\{v\}}=\sum_{i=1}^{v} z_{i}+2 \sum_{i=1}^{v} \sum_{j>i}^{v} \sqrt{z_{i} z_{j}} \cos \left(\phi_{i}-\phi_{j}\right) \tag{6}
\end{equation*}
$$

When the network traffic is light, the CSMA protocol introduces a random back-off mechanism that makes the probability of collisions involving three or more nodes negligible. We consider only collisions between two radio sources. Defining $\phi_{i j}=\phi_{i}-\phi_{j}$, Eq. (6) becomes

$$
\begin{equation*}
Z_{i j}=z_{i}+z_{j}+2 \sqrt{z_{i} z_{j}} \cos \left(\phi_{i j}\right) \tag{7}
\end{equation*}
$$

Since $\phi_{i j} \sim U[0,2 \pi]$, the cumulative distribution function (CDF) of $Z_{i j}$ is obtained as

$$
\begin{align*}
F_{Z}(y) & =p\left(Z_{i j} \leq y\right)=P\left(\cos \left(\phi_{i j}\right) \leq \frac{y-z_{i}-z_{j}}{2 \sqrt{z_{i} z_{j}}}\right) \\
& = \begin{cases}1 & \text { if } y \geq z_{i}+z_{j}+2 \sqrt{z_{i} z_{j}} \\
0 & \text { if } y \leq z_{i}+z_{j}-2 \sqrt{z_{i} z_{j}} \\
1-\frac{1}{\pi} \cos ^{-1}\left(\frac{y-z_{i}-z_{j}}{2 \sqrt{z_{i} z_{j}}}\right) & \text { otherwise. }\end{cases} \tag{8}
\end{align*}
$$

Since $z_{i}$ and $z_{j}$ are readings for individual receptions from radio sources $i$ and $j$, respectively, they can be obtained based on known radio source and robot locations using Eq. (2). Hence, we can compute the probability distribution of $Z^{k}=Z_{i j}$ over the integer reading $y$ when transmissions have collided,

$$
\begin{align*}
p\left(Z^{k}=y \mid i, j, \mathbf{x}_{k}\right) & =P\left(y-0.5 \leq Z^{k} \leq y+0.5\right) \\
& =F_{Z}(y+0.5)-F_{Z}(y-0.5) \tag{9}
\end{align*}
$$

## B. Sensing model

Now we are ready to compute the sensing model $p\left(Z^{k} \mid \mathbf{x}_{k}\right)$. Only three possibilities exist in the network, namely, no radio transmission, single transmission, and collision. When there is no transmission, $p\left(Z^{k}=0\right)=P_{\text {idle }}$, where the calculation of the channel idle probability $P_{\text {idle }}$ will be discussed in the next section. Considering that collisions involving three or more radio transmitters have a very small probability of occurring under light network traffic, we approximate the general sensing model by only considering two-source collisions,
$p\left(Z^{k} \mid \mathbf{x}_{k}\right)=\sum_{i=1}^{m} p\left(Z^{k} \mid i, \mathbf{x}_{k}\right) p(i)+\sum_{i=1}^{m} \sum_{j>i}^{m} p\left(Z^{k} \mid i, j, \mathbf{x}_{k}\right) p(i, j)$,
where $p(i)$ and $p(i, j)$ are the probabilities of the given collision types (no collision or collision) when the channel is busy, namely, non-zero received signal strength. We next discuss the calculation of $p(i)$ and $p(i, j)$.

## C. CSMA-based MAC protocol model

Let $P_{b}$ denote the channel busy probability, $P_{c}$ denote the channel collision probability, $P_{p c}$ denote the probability that a collision happens during a busy period, and $P_{b c}$ denote the long-run probability of collision given the channel is busy.

We compute $p(i)$ and $p(i, j)$ in Eq. (10) based on the CSMA-based MAC protocol model. Assuming that all the radio sources are identical and the communication protocol is fair, the probability that each radio source transmits is thus equal, and the probability that any two radio sources transmit during the same transmission period is equal. We define $p(i)=p, p(i, j)=q, i=1, \ldots, m, j>i$. Recall that $P_{\text {idle }}$ is the channel idle probability. The channel busy probability is then $P_{b}=1-P_{\text {idle }}$. We define the busy collision probability $P_{b c}$ as the conditional probability of a collision given the channel is busy, namely, $P_{b c}=\frac{P_{c}}{P_{b}}=\frac{P_{c}}{1-P_{i d l e}}$, where $P_{c}$ is the unconditional channel collision probability. The busy period is composed of the single-source sending and the two-source sending cases. Thus,

$$
\begin{equation*}
\sum_{i=1}^{m} p(i)+\sum_{i=1}^{m} \sum_{j>i}^{m} p(i, j)=m p+\frac{m(m-1)}{2} q=1 \tag{11}
\end{equation*}
$$

where $\sum_{i=1}^{m} \sum_{j>i}^{m} p(i, j)$ is the busy collision probability, and $P_{b c}=m(m-1) q / 2$. Therefore,

$$
\begin{equation*}
p=\frac{1}{m}\left(1-P_{b c}\right), \quad q=\frac{2 P_{b c}}{m(m-1)} \tag{12}
\end{equation*}
$$

If we know $P_{b c}$, we can compute $p$ and $q$. In addition, the measured channel idle probability $P_{i d l e}$ is critical to estimate the number of radio sources $m$. We now begin modeling for $P_{b c}$ and $P_{i d l e}$.


Fig. 5. CSMA: transmission period analysis.
Fig. 5 illustrates the timing diagram of a CSMA protocol. The time axis is divided into busy and idle periods alternatively. In the figure, $a \ll 1$ denotes the propagation delay, $t$ is the starting time of a busy period, and $t+Y$ is the time when the last packet arrives between $t$ and $t+a, 0<Y \leq a . B, I$, and $U$ are the durations of the busy period, the idle period, and the time during a cycle that the channel is used without conflicts, respectively. Each busy period is also termed a transmission period, which is further classified as a successful transmission period or an unsuccessful transmission period.

Without loss of generality, we set packet length $T=1$ in Fig. 5. A packet takes additional time $a$ to propagate. Therefore, a successful transmission takes time $(1+a)$. If a radio source decides to transmit Packet 2 at time $t$, then the duration between $t$ and $t+a$ is a "vulnerable" period because other radio sources cannot sense its transmission and may initiate another transmission (e.g. Packet 3), which leads to a collision.

We assume that each radio source is an independent Poisson source with the same packet generation rate $\lambda$. Thus, the aggregated transmission rate $S$ is given by $S=m \lambda$. Due
to retransmission (with rate $R$ ), the actual packet arrival rate $G=S+R$, and we call it the offered traffic rate. By the aggregation of several Poisson signal sources, $S$ is also a Poisson process. The offered traffic rate $G$ can also be approximated with a Poisson process. The collision probability of a transmission period $P_{p c}$ is calculated as $P_{p c}=1-e^{-a G}$. Upon each collision, there are two retransmissions scheduled. Each retransmission time is determined by the congestion backoff (CBO) value of the protocol used by the radio sources. Defining $\delta$ as the expected CBO time, $R$ can be represented as $R=2 S P_{p c}\left(\frac{1}{\delta}\right)=\frac{2 S\left(1-e^{-a G}\right)}{\delta}$. Therefore,

$$
\begin{equation*}
G=S+\frac{2 S\left(1-e^{-a G}\right)}{\delta} \tag{13}
\end{equation*}
$$

Now we are ready to compute the channel idle probability $P_{\text {idle }}$ and the busy collision probability $P_{b c}$. Using the renewal reward theorem and recalling that $B$ and $I$ are the length of the busy and idle periods, respectively, then $P_{\text {idle }}=\frac{\bar{I}}{\bar{B}+\bar{I}}$, where $\bar{I}=1 / G$ is the expected idle period, and $\bar{B}$ is the expected busy period. Recall that $Y$ is the moment that a packet arrives between $t$ and $t+a$ (Fig. 5). Then $\bar{B}=1+a+\bar{Y}$, where $\bar{Y}=E(Y)$ is the mean value of $Y$. The value of $Y$ depends on whether there is a collision or not. A collision means there is at least one new arrival in $[t, t+a)$. $Y=0$ if there is no collision. Therefore, we obtain $\bar{Y}=E(Y)=E(Y \mid$ collision $) P_{b c}=$ $E(Y \mid$ collision $)\left(1-e^{-a G}\right)$. The CDF of $Y$ given a collision case is

$$
\begin{aligned}
& F_{Y}(\tau \mid \text { collision })=p(Y \leq \tau \mid \text { collision }) \\
& =\frac{p(\text { no arrival in }[t+\tau, t+a), \text { at least one in }[t, t+a))}{p(\text { at least one in }[t, t+a))} \\
& =\frac{p(\text { at least one arrival in }[t, t+\tau))}{p(\text { at least one arrival in }[t, t+a))}=\frac{1-e^{-\tau G}}{1-e^{-a G}},
\end{aligned}
$$

where $0 \leq \tau<a$. Since $Y$ is non-negative,

$$
E(Y \mid \text { collision })=\int_{0}^{a}\left(1-F_{Y}(\tau \mid \text { collision })\right) d \tau
$$

We then obtain $\bar{Y}=\frac{1}{G}\left(1-e^{-a G}\right)-a e^{-a G}$ and

$$
\begin{equation*}
P_{i d l e}=\frac{1}{G(1+a)+2-e^{-a G}-a G e^{-a G}} \tag{14}
\end{equation*}
$$

The busy collision probability $P_{b c}$ is different from $P_{p c}$, since $P_{p c}$ is defined as the probability that a collision happens in a busy period, while $P_{b c}$ counts the total time that the channel has more than one simultaneous transmitter during a busy period. Since we consider the case that two radio transmissions collide (Fig. 5), the actual busy collision time should be $1-\bar{Y}$, and the actual busy period should be $1+$ $a+\bar{Y} . P_{b c}$ is obtained

$$
\begin{equation*}
P_{b c}=P_{p c} \frac{1-\bar{Y}}{1+a+\bar{Y}}=\left(1-e^{-a G}\right) \frac{1-\bar{Y}}{1+a+\bar{Y}} . \tag{15}
\end{equation*}
$$

Recalling that $S=m \lambda$, Eq. (13) establishes the connection between $m$ and $G$. We thus obtain $m$ using $P_{\text {idle }}$ based on Eqs. (13) and (14). We compute $p$ and $q$ using $m$ and $P_{b c}$ using Eq. (12). Hence, the sensing model in Eq. (10) is complete.

## IV. Algorithm and Experiments

## A. Localization algorithm

We summarize the algorithm as follows.

| Localization $\leftarrow$ false; step $\leftarrow 0 ; i_{\text {next }} \leftarrow 1$ | $O(1)$ |
| :--- | :--- |
| While Localization = false and step $<$ MAXSTEP do |  |
| Sample signal strength | $O(1)$ |
| Update $P_{i d l e}$ according to Eq. (14) | $O(1)$ |
| Update $Z^{k}$ according to non-zero signal readings | $O(1)$ |
| Update $m$ estimate according to Eqs. (13) and (14) | $O(1)$ |
| Compute $p\left(Z^{k} \mid \mathbf{x}_{k}\right)$ according to Eq. (10) | $O\left(m^{2}\right)$ |
| Apply SIS particle filter [14] with $n$ particles | $O(n m)$ |
| Extract next estimated radio source location with the |  |
| highest probability from particles | $O(n m)$ |
| Generate motion command and update robot position | $O(1)$ |
| Update $P_{b c}$ according to Eq. (15) | $O(1)$ |
| If a radio source is identified then |  |
| $\quad i_{\text {next }} \leftarrow i_{\text {next }}+1$ |  |
| Regenerate particles and update particle filter | $O(1)$ |
| If $i_{\text {next }}=m$ or step $>$ MAXSTEP then | $O(n m)$ |
| Localization $\leftarrow$ true | $O(1)$ |
| End if step $\leftarrow$ step +1 | $O(1)$ |
| End while |  |

Since the number of particles $n \gg m$, we know that the overall localization algorithm takes $O(n m)$ time.

## B. Experiments

We have implemented the localization algorithm using Microsoft Visual C++ .NET 2003 on a PC laptop with a 1.6 GHz Pentium-M CPU and 512MB RAM.

We first verify our CSMA modeling by comparing our model of $P_{i d l e}$ and $P_{b c}$ with the network simulation results (also implemented in Visual C++). We simulate the CSMA protocol for different values of $m, a$, and $\lambda$. The results in Fig. 6 show that our model conforms to the output of the network simulation when $\lambda$ is small. Because we assume light traffic in our model, large values of $\lambda$ mean heavy traffic and the probability of collisions involving more than two nodes increases. This is what we expected because we used the approximation in Eq. (10). The number of radio sources $m$ and the length of propagation delay $a$ have a similar effect on the model.


Fig. 6. Comparison of $P_{\text {idle }}$ and $P_{b c}$ from the network simulation and the proposed model under different values of $a, m$, and $\lambda$.

We then test the overall localization algorithm. We repeat the experiments for different network sizes $m$. Each data point in Fig. 7 is an average of 5000 independent trials with each radio source uniformly distributed in a square 2 D free space. The network has $a=0.02$ and $\lambda=0.05$, which is unknown to the robot. Initially 5000 particles are used in the particle filter. We are interested in how efficient the method


Fig. 7. The number of steps needed to localize an $m$-node network.


Fig. 8. Sample robot trajectory with network size $m=3$.
can localize the radio sources. Fig. 7 reports that the number of steps needed is linearly proportional to the network size, which conforms to our analysis. However, there are some outlying cases. It is unclear why the number of steps drops for $m=7$.

Fig. 8 illustrates a sample robot trajectory with a network size $m=3$. The parameters are the same as those in Fig. 7. As we can see, the density of the trajectory depends on how difficult it is to identify the radio sources. The two radio sources at the bottom are very close to each other. The robot has to wander around the region for a long time to obtain enough data to distinguish them.

## V. Conclusion and Future Work

We report our initial algorithmic developments of using a mobile robot equipped with a directional antenna to localize unknown networked radio sources. We propose a particle filter-based localization approach that combines a new sensing model with a CSMA model and a new directional antenna model. The new sensing model is capable of dealing with scenarios when transmission collisions occur. The overall algorithm runs in $O(n m)$ time for $n$ particles and $m$ radio sources at each step. We have implemented and tested the algorithm using a simulation platform. The results show that the algorithm is capable of localizing unknown networked radio sources when network traffic is light.

We are currently testing our algorithm using physical experiments. The particle filter has also shown its low efficiency
when the problem space becomes large. Therefore, we plan to develop other more scalable mechanisms.

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