Localization of Multiple Unknown Transient Radio Sources using Multiple Paired Mobile Robots with Limited Sensing Ranges

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***Abstract*— We develop a localization method enabling a team of mobile robots to search for multiple unknown transient radio sources. Due to signal source anonymity, short transmission durations, and dynamic transmission patterns, robots cannot treat the radio sources as continuous radio beacons. Moreover, robots do not know the source transmission power and have limited sensing ranges. To cope with these challenges, we pair up robots and develop a sensing model using the signal strength ratio from the paired robots. We formally prove that the sensed conditional joint posterior probability of source locations for the $m-$robot team can be obtained by combining the pairwise joint posterior probabilities, which can be derived from signal strength ratios. Moreover, we propose a pairwise ridge walking algorithm (PRWA) to coordinate the robot pairs based on the clustering of high probability regions and the minimization of local Shannon entropy. We have implemented and validated the algorithm under hardware-driven simulation.**

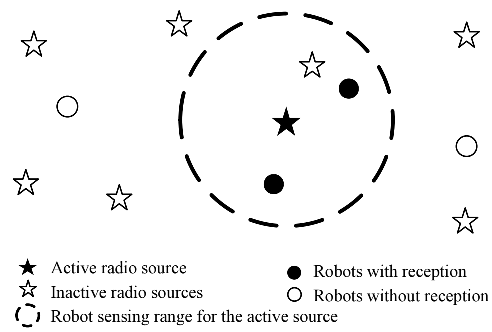

Fig. 1. An example of the localization scenario.

I. Introduction

Imagine that a team of mobile robots is searching for a sensor network deployed by enemies (see Fig. 1). The robots have little information about the sensor network except the fact that the sensor nodes emit short radio signals from time to time. Without knowledge of the network configuration and packet structure, localizing each node is difficult due to signal source anonymity, short transmission durations, and dynamic/intermittent transmission patterns. The robots can only rely on radio signal strength (RSS) from intercepted signals. However, the transmission power of the radio sources is unknown and may vary from time to time. A new method is needed for this multi-source localization problem that is coupled with issues in signal correspondence, variable source transmission power, and robot sensing range limits.

The recent development of radio frequency-based localization can be viewed as the localization of "friendly" radio sources because researchers either assume that an individual radio source continuously transmits radio signals (similar to a lighthouse) [1]–[4], or assume that robots/receivers are a part of the network and understand the detailed packet information [5]–[7]. However, such information is not always available for an unknown network. When signal sources are not cooperative, RSS readings are the primary information for localization because RSS attenuates over distance. Since signal transmission power at the source is not available, ratios between RSS readings from dislocated listeners have been proven to be effective [8]–[10]. Li et al. [11] shows that at least four robots are needed at the same moment in order to localize a single source with unknown transmission power. Another approaches use antenna arrays to obtain bearing readings. Kim and Chong [12] show how to find a radio source using two antennas with different polarizations. These approaches focus on single source localization and hence are not concerned with the signal correspondence issue.

Realizing that localizing unknown transient radio sources is an important new problem, our group studies the problem under different setups and constraints. First, we assume a carrier sense multiple access based protocol is used among networked radio sources [13], [14] which allows us to develop a particle filter-based approach. Then, we relax the assumption and develop a protocol-independent localization scheme using a spatiotemporal probability occupancy grid (SPOG) [15]. Our recent work [16], [17] find that teamed robots are more efficient than a single robot when the target is transient under the same sensing coverage. That result shifts our attention to the multi-robot based approach in this paper.

The contributions of this paper are twofold. First, we formally prove that the sensed conditional joint posterior probability of source locations for the $m-$robot team can be obtained by combining that of pairwise joint posterior probabilities, which are based on RSS ratios and also consider reception range limits. The new sensing model can be combined with the SPOG in [15] to address signal correspondence issue. Second, we propose a pairwise ridge walking algorithm (PRWA) to coordinate robot pairs based on the clustering of high probability regions and the minimization of local Shannon entropy. We have implemented and validated the algorithm under a hardware-driven simulation.

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II. Problem Definition

A. Problem Scenario

Both robots and radio sources reside in a 2D Euclidean space. We also make the following assumptions:

1) Each robot is equipped with an omni-directional antenna with a limited sensing range.
2) All robots are coordinated using a centralized control.
3) The unknown network traffic is light and each target radio transmission is short, which are the typical characteristics of a low power sensor network.
4) Transmission powers of radio sources are unknown to the robots and may change from time to time. However, locations of radio sources do not change.

B. Spatiotemporal Probability Occupancy Grid

To infer the transmitter locations and transmission rates based on perceived signals, we use a Bayesian framework to keep track of the knowledge of unknown radio sources. Here we extend the SPOG proposed in our previous work [15]. SPOG partitions the searching region into small and equal-sized grid cells. Define $i \in \mathcal{N}_i$ as the cell index variable where $\mathcal{N}_i := \{1, ..., n\}$ is the grid cell index set and n is the total number of cells. SPOG tracks two types of probabilistic events: C_i represents the event that cell i contains a radio source and C_i^1 represents the event that cell i is the active source when a transmission is detected. Define $P(C)$ as the probability for event C. $P(C_i)$ and $P(C_i^1)$ characterize spatiotemporal behaviors of transient radio sources. Note that we ignore collision cases because robots sense the radio signal strength (RSS) as soon as the transmission is initiated and the probability of two or more transmissions initiated at the exact same moment is negligible in a light traffic network.

Let $l \in \mathcal{M} := \{1, ..., m\}$ be the robot index variable where m is the total number of robots and $\mathcal{M}$ is the robot index set. Discrete time k refers to each moment when a transmission is detected by robots. Let the discrete random variable $\tilde{Z}_l^k \in [1, 255] \cap \mathbb{N}$ be the sensed RSS reading (from an 8-bit receiver) of the l-th robot at time k. Define $\tilde{\mathbf{Z}}^k = [\tilde{Z}_1^k, ..., \tilde{Z}_m^k]^T$ as a discrete random vector of all the sensed RSS readings at time k and let $\tilde{\mathbf{z}}^k := [\tilde{z}_1^k, ..., \tilde{z}_m^k]^T$ be corresponding values. As a convention, we use lower cases of random variables or vectors to denote their values.

At time k, event $\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k$ is perceived by robots. The posterior probability $P(C_i|\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k)$ over the grid needs to be updated. According to [15], this is actually a nested multivariate Bayesian process,

$$P(C_i|\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k) = \frac{\begin{pmatrix} P(\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k|C_i^1)P(C_i^1) + \\ P(C_i)\sum_{s\neq i, s\in I} P(\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k|C_s^1)P(C_s^1) \end{pmatrix}}{\sum_{i\in I} P(\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k|C_i^1)P(C_i^1)}, \quad (1)$$

$$P(C_i^1|\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k) = \frac{P(\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k|C_i^1)P(C_i^1)}{\sum_{i\in I} P(\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k|C_i^1)P(C_i^1)}, \quad (2)$$

where $P(\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k|C_i^1)$ is the sensing model. Eqs. (1) and (2) can be easily modified to an incremental conditional format for recursive update [15]. As more RSS readings enter the system over time, $P(C_i|\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k)$ converges and allows robots to localize each radio source.

C. Problem Formulation

To utilize the Bayesian framework, we need to derive a sensing model first:

Definition 1 (Sensing Problem): Derive $P(\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k|C_i^1)$ for present time k when a new RSS reading is received.

Once $P(\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k|C_i^1)$ is obtained, we can use (1) and (2) to compute posterior sensor location distribution $P(C_i|\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k)$, which leads to robot trajectory planning,

Definition 2 (Planning Problem): Given the updated $P(C_i|\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k)$, plan trajectories for each robot at the beginning of each planning period.

We start with the sensing problem first in Section III.

III. Sensing Model

The sensor model $P(\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k|C_i^1)$ is very complex. It is a joint conditional distribution of an m-dimensional random vector. To derive the conditional probability, we model the signal transmission uncertainty, derive pairwise sensing model based on signal strength ratio to remove the dependence on source transmission power, and propose a sensing fusion scheme to aggregate the output of all pairs to obtain the high order model $P(\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k|C_i^1)$. For simplicity, the time superscript k is dropped in this section by assuming that all values correspond to present time k. Thus, $P(\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k|C_i^1)$ becomes $P(\tilde{\mathbf{Z}} = \tilde{\mathbf{z}}|C_i^1)$.

A. Signal Propagation Model

For a robot equipped with an omni-directional antenna, the distance to the active radio source and source transmission power largely determine the perceived RSS. Assume the active radio source is located at the center of cell i. Let $\mathbf{x}_i = [x_i, y_i]^T$ and $\mathbf{x}_l = [x_l, y_l]^T$ be the center location of cell i and the location of robot l, respectively, when the transmission is sensed. Define $d_{li} = \| \mathbf{x}_l - \mathbf{x}_i \|$ as the Euclidean distance between $\mathbf{x}_l$ and $\mathbf{x}_i$. Following the signal propagation model [18], the expected RSS of robot l is denoted as ψ_l and measured in units of dBm:

$$\psi_l = w_i - 10\beta \log_{10}(d_{li}), \quad (3)$$

where source power level w_i is unknown and β is the signal decay factor.

An RSS level is not a constant but a continuous random variable due to uncertainties in transmissions. Assume the robot radio listener has an infinite resolution, its perceived RSS would be a continuous random variable Z_l for robot l. Moreover, robots can only detect the transmission signal if an active radio source is located in their sensing ranges, each of which is determined by an RSS threshold denoted by ζ. To characterize sensing range limit and background noises in sensing, we have

$$Z_l = \mu_l + \omega_l, \text{ where } \mu_l = \begin{cases} \psi_l, & \text{if } \tilde{z}_l > \zeta \\ \zeta, & \text{otherwise,} \end{cases} \quad (4)$$

where ω_l follows the independent and identically distributed (*i.i.d.*) Gaussian with zero mean and a variance of σ^2. Note that β in (3) and σ^2 can be obtained by calibration. Therefore, the probability density function (PDF) of $Z_l|C_i^1$ is $f_{Z_l|C_i^1}(z_l) = Bel(\mu_p, \sigma^2)$, where $Bel(\mu_p, \sigma^2)$ is the Gaussian PDF. As a convention, the subscript of $f(\cdot)$ is the corresponding random variable of the PDF function.

Actually, the sensed RSS reading $\tilde{Z}_l$ is an integer due to receiver hardware limit. As a convention, we use $\tilde{a}$ to indicate the integer value of continuous variable a. Define $\mathcal{I}_l$ as an RSS interval,

$$\mathcal{I}_l = (\tilde{z}_l - 0.5, \tilde{z}_l + 0.5] \subset \mathbb{R}. \quad (5)$$

Thus, we have the relation between $\tilde{Z}_l$ and Z_l given C_i^1,

$$P(\tilde{Z}_l = \tilde{z}_l|C_i^1) = P(Z_l \in \mathcal{I}_l|C_i^1) = \int_{z_l \in \mathcal{I}_l} f_{Z_l|C_i^1}(z_l) dz_l. \quad (6)$$

This is actually the sensing model when there is only one robot. Since this model relies on unknown source power level w_i, it is not a viable sensing model, but provides a foundation for the next step.

B. Transmission Power Independent Pairwise Sensing

For a robot pair (p, q), $p \neq q$, recall the possible RSS readings form sets $\mathcal{I}_p$ and $\mathcal{I}_q$ as defined in (5), respectively. According to our convention, $P(Z_p \in \mathcal{I}_p, Z_q \in \mathcal{I}_q|C_i^1)$ is a pairwise conditional probability given C_i^1. We are now ready to show that $P(Z_p \in \mathcal{I}_p, Z_q \in \mathcal{I}_q|C_i^1)$ can be obtained from its RSS ratio regardless of source transmission power levels.

Define $Z_{p-q} := Z_p - Z_q$ and let $\mathcal{I}_{p-q} = (\tilde{z}_p - \tilde{z}_q - 1, \tilde{z}_p - \tilde{z}_q + 1] \subset \mathbb{R}$ be the interval of Z_{p-q} values. $P(Z_{p-q} \in \mathcal{I}_{p-q}|C_i^1)$ denotes the probability of pairwise difference given C_i^1. Due to space limit, we have the following Lemma with its proof in our online technical report [19] supplementing this paper.

Lemma 1:

$$P(Z_p \in \mathcal{I}_p, Z_q \in \mathcal{I}_q|C_i^1) = \frac{1}{\eta_{pq}} P(Z_{p-q} \in \mathcal{I}_{p-q}|C_i^1), \quad (7)$$

where η_{pq} is the normalizing factor.

It is worth noting, since the RSS readings are in log scale, the difference between the two readings Z_{p-q} actually means a RSS ratio which does not depend on source transmission power levels. Computing $P(Z_{p-q} \in \mathcal{I}_{p-q}|C_i^1)$ is nontrivial because some of robots may not have readings due to limited sensing ranges. Based on (4), the robot index set $\mathcal{M}$ is partitioned into two disjoint sets $\mathcal{M} = \mathcal{M}_1 \cup \mathcal{M}_0$ which correspond to the sets of robots with and without receptions, respectively. As a result, we have three types of pairs: no detection for either robot, single detection, and dual detection. Define $\mathcal{E}$ as the set for all possible pairs which consists of three disjoint subsets $\mathcal{E} = \mathcal{E}_{11} \cup \mathcal{E}_{10} \cup \mathcal{E}_{00}$ where

$$\begin{aligned} \mathcal{E}_{11} &= \{(p,q)|p < q, p \in \mathcal{M}_1, q \in \mathcal{M}_1\}, \\ \mathcal{E}_{10} &= \{(p,q)|p \in \mathcal{M}_1, q \in \mathcal{M}_0\}, \\ \mathcal{E}_{00} &= \{(p,q)|p < q, p \in \mathcal{M}_0, q \in \mathcal{M}_0\}. \end{aligned} \quad (8)$$

Define Z_{p-q}^{11}, Z_{p-q}^{10} and Z_{p-q}^{00} as the sensor readings of the robot pair (p, q) corresponding to components of $\mathcal{E}_{11}$, $\mathcal{E}_{10}$ and $\mathcal{E}_{00}$, respectively. Z_{p-q} in (7) will be one of these three types. Note that $P(Z_{p-q}^{00} \in \mathcal{I}_{p-q}|C_i^1)$ is a constant because it provides no information due to no reception. We now focus on deriving $P(Z_{p-q}^{11} \in \mathcal{I}_{p-q}|C_i^1)$ and $P(Z_{p-q}^{10} \in \mathcal{I}_{p-q}|C_i^1)$.

Let us compute $P(Z_{p-q}^{11} \in \mathcal{I}_{p-q}|C_i^1)$ first. From (3) and (4), the mean value $(\mu_p - \mu_q)$ of Z_{p-q}^{11} becomes

$$\mu_p - \mu_q = \psi_p - \psi_q = 10\beta \log_{10} \frac{d_{qi}}{d_{pi}}, \quad (9)$$

and the PDF of $Z_{p-q}^{11}|C_i^1$ is

$$f_{Z_{p-q}^{11}|C_i^1}(z_{p-q}^{11}) = Bel\left(10\beta \log_{10} \frac{d_{qi}}{d_{pi}}, 2\sigma^2\right). \quad (10)$$

Thus, we have the following lemma.

Lemma 2:

$$\begin{aligned} P(Z_{p-q}^{11} \in \mathcal{I}_{p-q}|C_i^1) &= \int_{\tilde{z}_p - \tilde{z}_q - 1}^{\tilde{z}_p - \tilde{z}_q + 1} f_{Z_{p-q}^{11}|C_i^1}(z) dz \\ &= \left[\mathrm{F}_{Z_{p-q}^{11}|C_i^1}(\tilde{z}_p - \tilde{z}_q + 1) - \mathrm{F}_{Z_{p-q}^{11}|C_i^1}(\tilde{z}_p - \tilde{z}_q - 1)\right], \end{aligned} \quad (11)$$

where $\mathrm{F}_{Z_{p-q}^{11}|C_i^1}(\cdot)$ is the cumulative distribution function of $f_{Z_{p-q}^{11}|C_i^1}(\cdot)$.
To facilitate the understanding of the dual detection case, Fig. 2(a) shows an example to illustrate the corresponding posterior probability $P(C_i^1|Z_{p-q}^{11} \in \mathcal{I}_{p-q})$.

For $P(Z_{p-q}^{10} \in \mathcal{I}_{p-q}|C_i^1)$, we have the following result.

Lemma 3:

$$P(Z_{p-q}^{10} \in \mathcal{I}_{p-q}|C_i^1) = \frac{1}{\eta^{10}}\big(1 - \int_{\tilde{z}_p - \tilde{z}_q}^{\tilde{z}_p - \tilde{z}_q + 1} \mathrm{F}_{Z_{p-q}^{11}|C_i^1}(z) dz\big), \quad (12)$$

where η^{10} is the normalizing factor.
Again, the proof of Lemma 3 is in our technical report [19]. This result also does not depend on source transmission power. As an example, Fig. 2(b) illustrates the corresponding posterior probability $P(C_i^1|Z_{p-q}^{10} \in \mathcal{I}_{p-q})$.

C. Sensor Fusion of Multiple Pairs

Now we are ready to show that the m-dimensional joint conditional probability $P(\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k|C_i^1)$ can be reduced to a combination of pairwise conditional probabilities $P(Z_p \in \mathcal{I}_p, Z_q \in \mathcal{I}_q|C_i^1)$. We have the following lemma.

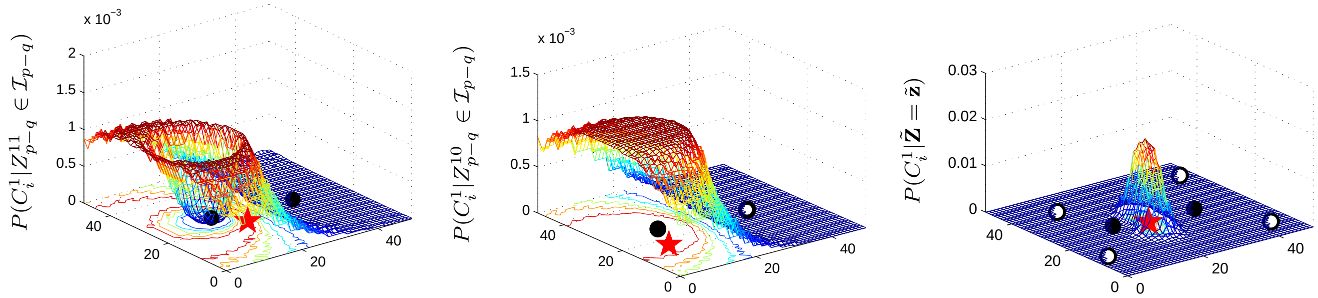
Lemma 4:

$$P(\tilde{\mathbf{Z}} = \tilde{\mathbf{z}}|C_i^1) = \frac{1}{\eta} \prod_{(p,q)\in\mathcal{E}} P(Z_p \in \mathcal{I}_p, Z_q \in \mathcal{I}_q|C_i^1), \quad (13)$$

where η is the normalizing factor and remains the same for all p and q values.
Again, the proof of Lemma 4 is in our technical report [19].

Now, we can complete the sensor model $P(\tilde{\mathbf{Z}} = \tilde{\mathbf{z}}|C_i^1)$. Combining Lemmas 1– 4, we have the following theorem.

Theorem 1: The high dimension joint conditional probability sensing model $P(\tilde{\mathbf{Z}} = \tilde{\mathbf{z}}|C_i^1)$ can be decomposed as a



(a) (b) (c)

Fig. 2. Sample cases of posterior condition distributions of signal source location given that $P(C_i^1)$ initially uniform across cells: (a) dual detection, (b) single detection, and (c) fusion of all pairs. The red star is the active radio source location. This is obtained using $P(Z_{p-q} \in \mathcal{I}_{p-q}|C_i^1)$ and the Bayesian framework in (2). The grid size is 50×50. Black and white dots represent robots with and without receptions, respectively.

combination of pairwise conditional probabilities,

$$P(\tilde{\mathbf{Z}} = \tilde{\mathbf{z}}|C_i^1) = \frac{1}{\eta''} \prod_{(p,q)\in\mathcal{E}_{11}} \begin{pmatrix} \mathrm{F}_{Z_{p-q}^{11}|C_i^1}(\tilde{z}_p - \tilde{z}_q + 1) \\ -\mathrm{F}_{Z_{p-q}^{11}|C_i^1}(\tilde{z}_p - \tilde{z}_q - 1) \end{pmatrix} \times \prod_{(p,q)\in\mathcal{E}_{10}} \Big(1 - \int_{\tilde{z}_p-\tilde{z}_q}^{\tilde{z}_p-\tilde{z}_q+1} \mathrm{F}_{Z_{p-q}^{11}|C_i^1}(z)dz\Big), \quad (14)$$

where η'' is the normalizing factor and remains the same for all p and q values.

Proof: Combining Lemma 1 with Lemma 4, the sensing model becomes

$$\begin{aligned} P(\tilde{\mathbf{Z}} =\tilde{\mathbf{z}}|C_i^1) &= \frac{1}{\eta} \prod_{(p,q)\in\mathcal{E}} \frac{1}{\eta_{pq}} P(Z_{p-q} \in \mathcal{I}_{p-q}|C_i^1) \\ &= \left(\frac{1}{\eta} \prod_{(p,q)\in\mathcal{E}} \frac{1}{\eta_{pq}}\right) \prod_{(p,q)\in\mathcal{E}} P(Z_{p-q} \in \mathcal{I}_{p-q}|C_i^1) \\ &= \frac{1}{\eta'} \prod_{(p,q)\in\mathcal{E}} P(Z_{p-q} \in \mathcal{I}_{p-q}|C_i^1), \quad (15) \end{aligned}$$

where $\eta' = \eta \prod_{(p,q)\in\mathcal{E}} \eta_{pq}$ is the normalizing factor and remains the same for all p and q values.

Applying (8) to (15) and combining Lemmas 2 and 3, the sensing model is rewritten as

$$\begin{aligned} P(\tilde{\mathbf{Z}} = \tilde{\mathbf{z}}|C_i^1) &= \frac{1}{\eta'} \prod_{(p,q)\in\mathcal{E}_{11}} P(Z_{p-q}^{11} \in \mathcal{I}_{p-q}|C_i^1) \\ &\times \prod_{(p,q)\in\mathcal{E}_{10}} P(Z_{p-q}^{10} \in \mathcal{I}_{p-q}|C_i^1) \\ &\times \prod_{(p,q)\in\mathcal{E}_{00}} P(Z_{p-q}^{00} \in \mathcal{I}_{p-q}|C_i^1) \\ &= \frac{1}{\eta''} \prod_{(p,q)\in\mathcal{E}_{11}} \begin{pmatrix} \mathrm{F}_{Z_{p-q}^{11}|C_i^1}(\tilde{z}_p - \tilde{z}_q + 1) \\ -\mathrm{F}_{Z_{p-q}^{11}|C_i^1}(\tilde{z}_p - \tilde{z}_q - 1) \end{pmatrix} \\ &\times \prod_{(p,q)\in\mathcal{E}_{10}} \Big(1 - \int_{\tilde{z}_p-\tilde{z}_q}^{\tilde{z}_p-\tilde{z}_q+1} \mathrm{F}_{Z_{p-q}^{11}|C_i^1}(z)dz\Big), \end{aligned} \quad (16)$$

where $\eta'' = \frac{\eta' \prod_{(p,q)\in\mathcal{E}_{10}} \eta^{10}}{\prod_{(p,q)\in\mathcal{E}_{00}} P(Z_{p-q}^{00} \in \mathcal{I}_{p-q}|C_i^1)}$ is the normalizing factor. ■

Again, Fig. 2(c) illustrates the corresponding posterior probability $P(C_i^1|\tilde{\mathbf{Z}} = \tilde{\mathbf{z}})$, which is the fusion of all pairs. It is desirable that the adjacent regions of the red star have higher probabilities than that of other regions.

IV. Robot Motion Planner

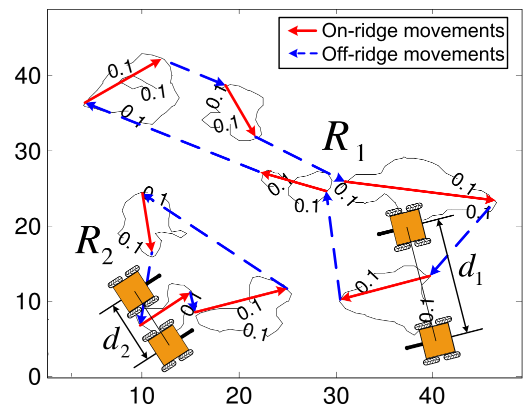


Fig. 3. An illustration of level sets with probability threshold of 0.1, ridges, and Pairwise Ridge Walking Algorithm with two pairs of robots over a 50×50 grid.

Theorem 1 summarizes how to compute $P(\tilde{\mathbf{Z}} = \tilde{\mathbf{z}}|C_i^1)$. With the sensing model, the Bayesian framework in (2) can derive the posterior source location distributions $P(C_i|\tilde{\mathbf{Z}} = \tilde{\mathbf{z}})$. The next step is to develop a multi-robot motion planner that enables robots to quickly localize radio sources using the SPOG. We build on the ridge walking algorithm (RWA) in [15]. RWA has been designed for a single robot without sensing range limit to localize multiple radio sources. The experimental results have shown that it is an efficient framework. However, RWA is not designed for multiple robots and significant revisions are needed. Let us begin with a brief review of RWA.

RWA uses a probability threshold plane that intercepts $P(C_i|\tilde{\mathbf{Z}} = \tilde{\mathbf{z}})$ to generate level sets that enclose all cells with $P(C_i|\tilde{\mathbf{Z}} = \tilde{\mathbf{z}})$ no less than the threshold. The irregular closed curves in Fig. 3 are examples of level sets. Ridges are created by extracting the longest dimension of each isolated level set. The directed red line segments in Fig. 3 are ridges. In RWA, a 3-opt heuristics algorithm is employed to compute an Euclidean traveling salesperson (TSP) tour for the single robot that must include all ridges. The TSP tour is partitioned into on-ridge and off-ridge segments. For off-ridge segments, the robot moves at its fastest speed. For on-ridge segments, the robot spends the time proportional to the summation of posterior conditional probability $P(C_i|\tilde{\mathbf{Z}} = \tilde{\mathbf{z}})$ over the corresponding isolated level set on each ridge. This means that the robot spends more time in high probability regions, which increases the localization efficiency.

Since we have more than one robot, we need many sub tours instead of a single TSP tour. We pair up robots and treat a pair of robots as a super robot. Assuming m is an even number, we have $m/2$ super robots. Therefore, we need to partition the TSP tour into $m/2$ sub tours and assign each super robot to a sub tour. The partition is based on k-means clustering algorithm [20] with $m/2$ as the cluster number to cluster ridge sets. For each cluster, we again use a 3-opt heuristics algorithm to find the TSP and the rest of RWA follows. Hence, we name this approach pairwise ridge walking algorithm (PRWA).

The remaining issue is how to determine the distance between each paired robots. Comparing Fig. 2(a) and (b), we notice that the dual detection case provides more information (less uncertainty) about radio source locations than the single detection case does. The spatial information contained in a distribution can be measured by the Shannon entropy in information theory. In order to choose the best distance d_u^* between the u-th pair, we formulate this problem by minimizing the Shannon entropy.

Define $\mathcal{S}_u$ as the set of cells in the isolated level set that correspond to the ridge cluster $\mathbf{R}_u$. Let cell $v \in \mathcal{S}_u$. Assume that the radio source $\mathbf{x}_v = [x_v, y_v]^T$ is located at the center of cell C_v by ignoring the minor intra-cell difference. Define $\hat{z}_{lv}^w$ as the mean RSS reading at robot l. We have,

$$\hat{z}_{lv}^w = w - 10\beta \log_{10}(d_{lv}), \tag{17}$$

where $w \in [w_{min}, w_{max}]$ is the unknown source transmission power which varies from w_{min} to w_{max}.

Define $\hat{\mathbf{Z}}_v^w = [\hat{Z}_{pv}^w, \hat{Z}_{qv}^w]^T$ as the RSS readings for the robot pair. Define $\mathbf{r}_u(t)$ as the center position of the robot pair at time t. We know $\mathbf{r}_u(t)$ because PRWA provides the trajectory for the super robot using the center position of the robot pair as the position on the trajectory. Denote $P(C_i|\hat{\mathbf{Z}}_v^w = \hat{\mathbf{z}}_v^w, \mathbf{r}_u(t), d_u)$ as the posterior probability that cell i contains a radio source given $\hat{\mathbf{z}}_v^w$, $\mathbf{r}_u(t)$ and d_u. Define $H(t, w, v, d_u)$ as the Shannon entropy over the probability distribution $P(C_i|\hat{\mathbf{Z}}_v^w = \hat{\mathbf{z}}_v^w, \mathbf{r}_u(t), d_u)$ given v, w and d_u. $H(t, w, v, d_u)$ is given by

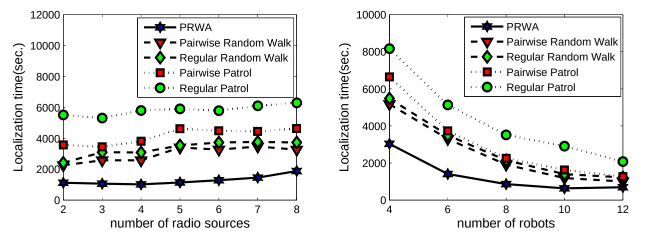
$$H(t, w, v, d_u) = -\sum_{i \in \mathcal{S}_u} \left(\begin{array}{l} P(C_i|\hat{\mathbf{Z}}_v^w = \hat{\mathbf{z}}_v^w, \mathbf{r}_u(t), d_u) \\ \times \ln P(C_i|\hat{\mathbf{Z}}_v^w = \hat{\mathbf{z}}_v^w, \mathbf{r}_u(t), d_u) \end{array} \right) \tag{18}$$

where $P(C_i|\hat{\mathbf{Z}}_v^w = \hat{\mathbf{z}}_v^w, \mathbf{r}_u(t), d_u)$ is obtained from (1) and (2) after calculating the sensing model (14) with $\hat{\mathbf{z}}_v^w$. We choose the optimal d_u^* that minimizes the following Shannon entropy for the cluster region over the period τ_u when the robot is inside $\mathbf{R}_u$,

$$d_u^* = \arg\min_{d_u} \int_t^{t+\tau_u} \sum_{w=w_{min}}^{w_{max}} \sum_{v \in \mathcal{S}_u} H(t, w, v, d_u). \tag{19}$$

Note that here we assume that w is evenly distributed over integer values in $[w_{min}, w_{max}]$. In fact, we can estimate the more accurate distribution of w once more received signals become available to improve the model.

V. Experiments



(a) (b)

Fig. 4. Experiment results: (a) Localization time vs. number of radio sources. (b) Localization time vs. number of robots.

We have implemented the algorithms and the simulation platform using Microsoft Visual C++ .NET 2005 with OpenGL on a PC Desktop with an Intel 2.13GHz Core 2 Duo CPU, 2GB RAM, and Windows XP. The radio sources are XBee Pro with ZigBeeT/802.15.4 OEM radio frequency modules produced by Digi International Inc. The antenna is calibrated first with the radio sources. The calibration establishes the parameters in (3). We use the data from the real hardware to drive the simulation experiments below.

The grid is a square with 50×50 cells. Each grid cell has a size of 50.0×50.0 cm^2. Each radio source generates radio transmission signals according to an *i.i.d.* Poisson process with a rate of $\lambda = 0.05$ packets per second. We choose the probability convergence threshold as $p_t = 0.9$ which means if $P(C_i|\tilde{\mathbf{Z}} = \tilde{\mathbf{z}}) > 0.9$, the algorithm outputs the cell as a radio source location. During each trial of the simulation, we randomly generate radio source locations in the grid and randomly set their power levels as one of five power levels offered by XBee Pro nodes.

We compare the PRWA algorithm to four heuristics. Two of the four heuristics are based on random walk: a pairwise random walk and a regular random walk. In the pairwise random walk, robots are paired just as PRWA does. Each pair is treated as a super robot to perform a random walk together while all robots perform independent movements in the regular random walk. The remaining two heuristics are based on a fixed-route patrol: the robots patrol the field using a predefined route that covers the search region. Again, robots are either paired which results in a pairwise patrol or non-paired which results in a regular patrol. Robot pairs in the pairwise patrol or individual robots in the regular patrol are distributed evenly along the route to increase coverage.

The experiment compares all five methods under different numbers of radio sources and robots. Figs. 4(a) and 4(b) illustrate experiment results. Each data point is an average of 100 independent trials. The results show that PRWA is consistently the fastest method under all comparisons. Also, the pairwise random walk and the pairwise patrol are consistently faster than the regular random walk and patrol, respectively. This is expected because paired robots are more efficient with their limited sensing ranges. Another interesting observation is that the two random walk-based methods are faster than the two fixed-route patrol methods. This is expected because random walk can bring robots together from time to time, which increases the number of effective pairs and hence listening efficiency. The fixed-route patrol methods emphasize coverage and spread robot pairs or individual robots apart along the route and hence cannot create many effective pairs, which decreases localization efficiency. The results in Fig. 4(b) also show that the difference between the five methods decreases as the number of robots increases. However, in reality, the number of robots is often constrained to where PRWA is superior.

VI. Conclusions and Future Work

We reported a new localization method that enables a team of mobile robots to localize multiple unknown transient radio sources. To cope with the challenges from signal correspondence, limited sensing ranges, and unknown transmission power, we paired up robots and developed a sensing model using the signal strength ratio from the paired robots. We formally proved that the sensed conditional joint posterior probability of source locations for the $m-$robot team can be obtained by combining that of pairwise joint posterior probabilities. Moreover, we proposed a pairwise ridge walking algorithm (PRWA) to coordinate the robot pairs based on the clustering of high probability regions and the minimization of local Shannon entropy. We implemented the algorithm and tested it under hardware-driven simulation. Results show that PRWA-based localization consistently outperforms the other four heuristics in all settings tested. We are currently testing our algorithm using physical experiments. Results will be reported in a subsequent journal version. In the future, we will address the decentralized control issue by proving that the joint posterior probability updating process can be handled locally in the distributed pairs. We will study how the information exchange rate between pairs affects convergence speed to provide theoretical bounds on searching time of distributed approaches.

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