Decentralized Searching of Multiple Unknown and Transient Radio Sources

Chang-Young Kim, Dezhen Song, and Jingang Yi

Abstract— **We develop a decentralized algorithm to coordinate a group of mobile robots to search for unknown and transient radio sources in an open field. In addition to their limited mobility and ranges of communication and sensing, the robot team has to deal with challenges from signal source anonymity, short transmission duration, variable transmission power, and the unknown source number since the radio sources may not be friendly. Building on our prior work, we propose a two-step approach: first, we decentralize belief functions that robots use to track source locations using checkpoint-based synchronization, and second, we propose a decentralized planning strategy to coordinate robots to ensure the existence of checkpoints. We formally analyze memory usage, data amount in communication, and the expected searching time for the proposed algorithm. We have implemented the proposed algorithm and compared it with two heuristics in simulation based on real sensory data. The experiment results show that our algorithm successfully trades a modest amount of memory for the fastest searching time among the three methods.**

I. INTRODUCTION

The fast development of wireless sensor network (WSN) technology provides great tools to collect information. However, WSNs can also be a significant threat to our security and privacy in hostile environments (e.g. an enemy may deploy a sensor field to detect troop movements). The large number of miniature sensors in a large field makes it difficult to manually search and neutralize the sensors. We are developing algorithms to enable a team of mobile robots to perform the task.

In this "robot network" vs. "sensor network" setup, each party has its own advantages and limitations. Robots have mobility while sensors do not. Robots know their own locations and received signal strength (RSS) readings. However, robots do not understand the network protocol of the sensor network and have to treat sensors as plain radio sources. Furthermore, signal source anonymity, short transmission duration, variable transmission power, and the unknown source number further challenge robots in addition to their communication and sensing range constraints.

Building on our prior work, we propose a two-step approach: first we decentralize belief functions that robots use to track source locations using checkpoint-based synchronization, and second we propose a decentralized planning strategy to coordinate robots to ensure the existence of checkpoints. We formally show that our planning algorithm

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ensures the decentralized belief functions to be synchronized periodically with explicit analysis on memory and communication requirements. Furthermore, the expected searching time of our algorithm is insensitive to the number of radio sources. We have implemented the proposed algorithm and compared it with two heuristics in simulation based on real sensory data. The experiment results show that our algorithm successfully trades a modest amount of memory for the fastest searching time among the three methods in comparison.

II. RELATED WORK

Searching for multiple transient radio sources under communication and sensing range constraints relates to the emerging fields of RSS-based localization, multi-robot motion estimation and planning.

The recent development of radio frequency-based localization can be viewed as the localization of "friendly" radio sources because researchers either assume that an individual radio source continuously transmits radio signals (similar to a lighthouse) [1], [2], or assume that robots/receivers are a part of the network and understand the detailed packet information [3], [4]. However, such information is not always available for an unknown network. When signal sources are not cooperative, RSS readings are the primary information for localization because RSS attenuates over distance. When signal transmission power at the source is not available, ratios between RSS readings from dislocated listeners [5], [6] or an antenna array [7] have been proven to be effective in obtaining bearing and/or range readings. These approaches focus on single source localization and hence are not concerned with the signal correspondence issue in the multi-source case.

In multi-robot research area, decentralized estimation of robot positions and poses has recently gained a lot of attention. Durrant-Whyte et al. [8]–[10] develop the decentralized estimation technology for the static sensor network using the information filter based on the distributed Kalman filter framework. Based on relative observations between multiple robots, researchers propose various decentralized multi-robots localization schemes by the decentralized extended Kalman filter (EKF) [11], which is decomposed into a number of smaller communicating filters, and by the decentralized maximum a posteriori (MAP) estimator [2]. Leung et al. [12] also tackle the decentralized multi-robot localization problem by a concept of the checkpoint, which is a delayed synchronization of observation after exchanging observations between robots. Researchers extend the checkpoint concept to Decentralized-Simultaneous Localization and Mapping (D-SLAM) [13] and the decentralized information transfer

scheme [14] based on communication resources available. Capitan et al. [15] also propose a delayed synchronization for decentralized estimation using the information filter. Most of the decentralized estimation methods focus on estimating robot positions and orientations with range or heading readings from onboard sensors. Our problem is similar in the way that we can benefit from the range and heading sensory models, but it is different because we focus on estimating positions of transient targets instead of robots themselves.

Also in multi-robots research, Pereira et al. [16] propose the decentralized planning under sensing and communication constraints while keeping connectivity with the neighbors. By using decentralized multi-robots, Bhadauria et al. [17] address the Data Gathering Problem (DGP) in which multiple robots gather information from deployed sensor networks. In this work, they formulate the DGP as Travel Salesman Problem (TSP) instances, and propose a two sub tour plans. One tour is a counterclockwise tour and the other is a clockwise tour to ensure that the two tours cover entire deployed sensor nodes. Another aspect in the decentralized planning is synchronization. Martinez et al. [18], [19] analyze motion synchronization of decentralized multi-robots introducing a network of locally connected agents on the tour using the agree-and-pursue algorithm. These works on communication and sensing constraints inspire our work. Unlike the popular pursuit-evasion game, radio sources in our problem do not move. However, the stationary nodes do not make the problem simpler because radio sources are transient and can change transmission power from time to time which forms a different type of problem.

Realizing that searching for unknown transient radio sources is an important new problem, our group studies the problem under different setups and constraints. First, we assume a carrier-sense-multiple-access based protocol is used among networked radio sources [20], which allows us to develop a particle filter-based approach. Then we relax the assumption and develop a protocol-independent Bayesian localization scheme [21], [22]. Our recent works [23], [24] find that teamed robots are more efficient than a single robot when the target is transient under the same sensing coverage. That results shift our attention to the multi-robot based approach in this paper. This paper differs from the centralized pairwise searching method in [25] by decentralizing belief functions and proposing a new decentralized planning algorithm.

III. PROBLEM DEFINITION

Our searching problem builds on the following setup and assumptions:

1) Both robots and radio sources reside in an open 2D Euclidean space.
2) Each robot has a limited communication range and a limited sensing range.
3) Each robot knows its position using the Global Positioning System (GPS). GPS clocks also provide accurate time for the synchronization purpose.
4) The unknown network traffic is light and each target

radio transmission is short, which are the typical characteristics of a low power sensor network.
5) Transmission powers of radio sources are unknown to robots and may change from time to time. However, locations of radio sources do not change.

For the new decentralized approach, we will follow the same problem definition in the corresponding centralized versions [21], [22], [25], where the searching problem is partitioned into two sub problems:

Definition 1 (Sensing Problem): Given the RSS readings and corresponding locations from robots, update robot belief functions for radio source locations.

Definition 2 (Planning Problem): Given the belief functions, plan robot trajectories to increase searching efficiency. We will concretely define the belief functions in detail later in the paper. As we can see, this is a Monte Carlo type algorithmic approach with the following stopping time for radio source detection,

Definition 3 (Searching Condition): A radio source is considered as found if the belief function is bigger than a preset threshold p_t.

Now let us begin with the sensing problem.

IV. DECENTRALIZED BELIEF FUNCTIONS

Belief functions track the radio source distribution based on RSS readings and robot locations. They are usually built on a Bayesian framework and antenna models to allow incremental update. In our previous work [21], [25] on the centralized localization of transient and unknown radio sources, we propose a Spatial Temporal Occupancy Grid (SPOG) as the robots' common belief functions. Let us review it first and then we will decentralize SPOG.

A. A Brief Review of SPOG

SPOG partitions the searching region into small and equal-sized grid cells. Define $i \in \mathcal{N}$ as the cell index variable where $\mathcal{N} := \{1, ..., n\}$ is the grid cell index set and n is the total number of cells. SPOG tracks two types of probabilistic events: C_i represents the event that cell i contains a radio source and C_i^1 represents the event that cell i is the active source when a transmission is detected. C_i^1 actually reflects the relative transmission rates among multiple sources, which is a temporal dimension signature. Define $P(C)$ as the probability for event C. $P(C_i)$ and $P(C_i^1)$ characterize spatiotemporal behaviors of transient radio sources.

Let $l \in \mathcal{M} := \{1, ..., m\}$ be the robot index variable where m is the total number of robots and $\mathcal{M}$ is the robot index set. Note that m is always an even number since we will pair robots up later. Discrete time k or the corresponding continuous time t^k refers to each moment when a transmission is detected by robots. Let $\mathbf{x}_l^k := [x_l^k, y_l^k]^T$ be the location of robot l at time k and $\mathbf{X}^k := [\mathbf{x}_1^k, ..., \mathbf{x}_m^k]^T$ be a set of all robot locations at time k. Let the discrete random variable $\tilde{Z}_l^k$ be the RSS reading of the l-th robot at time k. Define $\tilde{\mathbf{Z}}^k := [\tilde{Z}_1^k, ..., \tilde{Z}_m^k]^T$ as a discrete random vector of all the RSS readings at time k and let $\tilde{\mathbf{z}}^k := [\tilde{z}_1^k, ..., \tilde{z}_m^k]^T$ be corresponding values. As a convention, we use lower cases

of random variables or vectors to denote their values. Define $\mathcal{Z}^{1:k} := \{\tilde{\mathbf{z}}^1,...,\tilde{\mathbf{z}}^k\}$ as the set of all RSSs sensed from the beginning of the searching to t^k. Define $P(C_i|\mathcal{Z}^{1:k})$ as the conditional probability that cell i contains at least one radio source given $\mathcal{Z}^{1:k}$. Similarly, we define $P(C_i|\mathcal{Z}^{1:k-1})$, $P(C_i^1|\mathcal{Z}^{1:k})$, and $P(C_i^1|\mathcal{Z}^{1:k-1})$.

At time k, event $\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k$ is perceived by robots. The posterior probability $P(C_i|\mathcal{Z}^{1:k})$ over the grid needs to be updated. According to [21], this is actually a nested multivariate Bayesian process,

$$P(C_i|\mathcal{Z}^{1:k}) = \\ \frac{\begin{pmatrix} P(\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k|C_i^1)P(C_i^1|\mathcal{Z}^{1:k-1}) + \\ P(C_i|\mathcal{Z}^{1:k-1}) \times \\ \sum_{s \neq i, s \in \mathcal{N}} P(\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k|C_s^1)P(C_s^1|\mathcal{Z}^{1:k-1}) \end{pmatrix}}{\sum_{i \in \mathcal{N}} P(\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k|C_i^1)P(C_i^1|\mathcal{Z}^{1:k-1})}, \quad (1)$$

$$P(C_i^1|\mathcal{Z}^{1:k}) = \frac{P(\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k|C_i^1)P(C_i^1|\mathcal{Z}^{1:k-1})}{\sum_{s \in \mathcal{N}} P(\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k|C_s^1)P(C_s^1|\mathcal{Z}^{1:k-1})}, \quad (2)$$

where $P(\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k|C_i^1)$ refers to the conditional probability for event $(\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k)$ when cell i is transmitting. This is the sensing model. In [21], [22], we have shown how to obtain $P(\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k|C_i^1)$ from antenna radiation pattern and signal attenuation model. It is worth noting that $P(\tilde{\mathbf{Z}}^k = \tilde{\mathbf{z}}^k|C_i^1)$ has to be established on signal ratios from robot pairs when the source transmission power is unknown. Since this problem is detailed in the centralized version [25], we omit it here. It is worth noting that SPOG allows different antenna models and is flexible in different antennas or robot configurations. Eqs. (1) and (2) are in an incremental conditional format for recursive update. As more RSS readings enter the system over time, $P(C_i|\mathcal{Z}^{1:k})$ converges until $P(C_i|\mathcal{Z}^{1:k}) > p_t$ which means that searching condition in Def. 3 is satisfied.

B. Decentralized SPOG (D-SPOG):

In the decentralized system, each robot has to maintain its own local SPOG by accumulating RSS readings internally and exchanging information with other robots whenever other robots move into its communication range. However, the centralized SPOG described in (1) and (2) depends on the strict order of complete observation set $\mathcal{Z}^{1:k}$. Robots cannot arbitrarily use their partial receptions to generate a local SPOG. Furthermore, robots cannot keep their readings forever for future information exchange due to limited on-board memory space.

Before we address this problem, let us take a close look at the decentralized system. There are three types of discrete events in the decentralized system: *detection events* referring to moments when a transmission is detected by robots, *rendezvous events* describing moments when a robot moves into another robot's communication range, and *planning events* describing moments when a robot starts a new path planning. Recall that k is the time index variable for the detection event. Denote j and κ as the rendezvous event and the planning event, respectively. Define $t_\kappa^{j;k}$ to describe the

three events in the continuous time domain as a convention in the paper. To reduce cluttering, we may also use a reduced version such as t^k and t^j for the corresponding event time. t_κ indicates the beginning of the κ-th planning period.

An effective coordination plan should allow robots to exchange information among each other so that all robots have the same set of observations $\mathcal{Z}^{1:k}$ at time j, $t^j \geq t^k$. This is the time that all robots can update their SPOG up to time k. In such a way, the centralized SPOG can be decentralized and synchronized among all robots. The "delayed synchronization" concept is proposed as a checkpoint by Leung et al. [12]. Let us denote $Y(t^k, t^j)$ as the checkpoint. Note that each checkpoint for a robot always has two time variables: it begins with an early detection event time and ends with a future rendezvous event time because information is always generated by detection events and synchronized by rendezvous events.
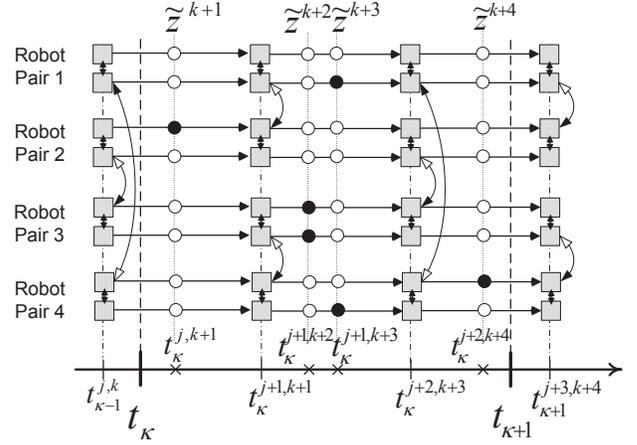

Fig. 1. A sample information flow graph for four robot pairs. Gray rectangles represent robot rendezvous events. Arc arrows in vertical direction indicate information exchange between robots in communication range. Black and white circles represent events for robots with and without detection of active radio transmissions, respectively.

Fig. 1 shows an information flow graph to illustrate the checkpoint concept and how information is passed around the distributed robot pairs in D-SPOG. Note that robots have been paired up in this graph because it takes two robots to obtain a signal ratio for radio sources with unknown and variable transmission powers. We ignore the intra-pair communication because a pair can always talk to each other according to planning. Following the arc arrows in vertical directions, we can see that both $Y(t_\kappa^{j,k+1}, t_\kappa^{j+2,k+3})$ and $Y(t_\kappa^{j+1,k+2}, t_\kappa^{j+3,k+4})$ are checkpoints.

To build a D-SPOG, the remaining question is how each robot stores and exchanges information. Say that $Y_l(t^{k-1}, t^{j-1})$ is the last checkpoint for robot l. After the update at t^{j-1}, D-SPOG for the robot l is synchronized up to t^{k-1} with the fictitious centralized SPOG according to the checkpoint property. Robot l only needs to store its own locations and RSS readings after t^{k-1}, which results in significant saving in memory. Due to the fact that robots without detection may not know the time of the radio

transmission, each robot has to keep track of its trajectory in addition to RSS readings. Let $\mathcal{W}_l^{k-1,t}$ be the measurement set internally generated by robot l between t^{k-1} and current time $t, t > t^{k-1}$:

$$\mathcal{W}_l^{k-1,t} = \left\{ \mathbf{x}_l((t^{k-1}, t]), \mathbf{z}_l((t^{k-1}, t]) \right\}, \qquad (3)$$

where $\mathbf{x}_l(\cdot)$ is the robot trajectory and $\mathbf{z}_l(\cdot)$ is the RSS reading set for the duration. Similarly, we define $\mathcal{W}_l^{k-1,j}$ and $\mathcal{W}_l^{k-1,k}$ by replacing t with t^j and t^k, respectively.

Let us define the measurement set of robot l at rendezvous time t^j as $\Gamma_l^{k-1,j}$ which contains information from both its on-board sensors and other robots. To describe the moment right before the robot l encounters another robot, we introduce a $(\cdot)^-$ notation. It is clear that $\mathcal{W}_l^{k-1,j} \subseteq (\Gamma_l^{k-1,j})^-$. At t^j, robot l meets robot p, which has measurement set $(\Gamma_p^{k'-1,j})^-$ prior to the information exchange where $t^{k'-1}$ is the detection event time of the last checkpoint that robot p has. Note that $t^{k'-1}$ and t^{k-1} are not necessarily the same. The two robots first compare the two times because a newer time means a more recent D-SPOG. The other robot should synchronize its SPOG to the recent one. After synchronizing their SPOGs, they need to synchronize the measurement set. Note that we have $(\Gamma_l^{k-1,j})^-$ for robot l and $(\Gamma_p^{k'-1,j})^-$ for robot p before the synchronization. Without loss of generality, we assume $t^{k-1} \geq t^{k'-1}$, the synchronization process is,

$$\Gamma_l^{k-1,j} = \Gamma_p^{k-1,j} = (\Gamma_l^{k-1,j})^- \cup (\Gamma_p^{k-1,j})^-, \qquad (4)$$

where $(\Gamma_p^{k-1,j})^- = (\Gamma_p^{k'-1,j})^- \setminus \Gamma_p^{k'-1,k-1}$ is obtained by discarding the measurement between $t^{k'-1}$ and t^{k-1}, a reduction in memory usage.

After the rendezvous event, each robot needs to search if a more recent checkpoint can be established. For robot l, it checks $\Gamma_l^{k-1,j}$ to see if the measurement set contains information from all other robots for detection events happened after the $k-1$-th detection event by searching for the maximum δ,

$$\delta = \arg \max_{\delta \in \mathbb{Z} \cap [-1, \infty) \text{ and } t^{k+\delta} \leq t} \delta \left[\Pi_{p=1}^m (\mathcal{W}_l^{k-1,k+\delta} \subset \Gamma_l^{k-1,j}) \right], \qquad (5)$$

where $(\mathcal{W}_l^{k-1,k+\delta} \subset \Gamma_l^{k-1,j})$ is a logic operation which returns 0 if the relationship is not satisfied and 1 otherwise. Only the existence of nonnegative solution indicates a new checkpoint $Y_l(t^{k+\delta}, t^j)$ can be established and hence the D-SPOG can be updated. After the update, it is clear that D-SPOG is equivalent to the centralized SPOG update with a delay of $t - t^{k+\delta}$. We have the following lemma,

Lemma 1: To ensure proper update of D-SPOG at checkpoints, both the amount of information that every robot stores onboard and the amount of information exchange during the rendezvous event between two robots are $O(n+m(t-t^{k-1}))$, where t is current time and t^{k-1} is the detection event time of the latest checkpoint.

Proof: Each robot has to store a D-SPOG which takes $O(n)$ memory space. At the worst case scenario, $\Gamma_p^{k-1,t}$ may contain $m-1$ robots' trajectories and RSS reading sets from

t^{k-1} to t. Since the trajectory storage using a fixed period and the mean number of transmissions is linear to the time duration, hence the overall amount of information stored on each robot is $O(n+m(t-t^{k-1}))$. Since both D-SPOG and their measurement sets need to be synchronized during the rendezvous of robots, the lemma holds. ∎

However, if just one robot is geographically isolated with others which results in no communication to others, no checkpoint can be established. $t - t^{k-1}$ becomes unbounded and the robot may quickly run out of memory which leads to failure. To address this problem, we propose a decentralized planning that guarantees periodic checkpoint existence.

V. DECENTRALIZED PLANNING

The decentralized planning strategy needs to take checkpoint existence, communication range limit, synchronization, and searching time into consideration. We build the new planning strategy on our existing ridge walking algorithm (RWA), which was designed for the centralized version of the problem in [21], [22]. RWA and its variants have been proved to be effective in accelerating the convergence of $P(C_i | \mathcal{Z}^{1:k})$ to reduce searching time and have excellent scalability.

A. A Brief Review of RWA and Pairwise RWA (PRWA)

In SPOG or D-SPOG, $P(C_i | \mathcal{Z}^{1:k})$ is the conditional probability that cell i contains a radio source. RWA plans a path for a single robot by building on this spatial distribution of radio sources. We generate a level set $L(p)$, $p \in (0, 1]$ by using a plane parallel to the ground plane to intersect the mountain-like distribution $P(C_i | \mathcal{Z}^{1:k})$ at height p. The intersection generates $L(p)$ which contains all cells with $P(C_i | \mathcal{Z}^{1:k})$ above the plane. $L(p)$ usually consists of several disconnected components. The irregular contours in Fig. 2(a) is an example of $L(0.1)$. For each component, we define its ridge as the longest line segment along its dominating direction [22]. We know each ridge has very high probability of being close to a potential signal source. We generate a Traveling Salesperson Problem (TSP) tour which contains all ridges. For off-ridge segments, the robot moves at its fastest speed. The solid red and dashed blue lines in Fig. 2(a) represent on-ridge and off-ridge movements, respectively. For on-ridge segments, the robot spends the time proportional to the summation of posterior conditional probability $P(C_i | \mathcal{Z}^{1:k})$ over the corresponding isolated level set on each ridge. This allows the robot to spend most of its time on ridges, then the intuition yields the ridge walking algorithm (RWA) as detailed in [22]. RWA has shown superior convergence performance and scalability in searching for multiple signal sources.

PRWA extends RWA to plan trajectories for a team of robots to handle unknown and changing transmission power in [25]. First, PRWA expands SPOG by developing a pairwise sensing model based on RSS ratios from robot pairs instead of assuming known absolute source transmission power. Second, PRWA coordinates robots in pairs by minimizing information entropy so that a robot pair can be viewed as a super-robot in planning. We will inherit the

pairwise sensing model and coordinate robots in pairs in this decentralized version.

B. Decentralized Pairwise Ridge Walking Algorithm (DPRWA)

Similar to RWA, DPRWA coordinate robot pairs to patrol on TSP tours that link all ridges. We generate a TSP tour in each planning period. In fact, each planning period is divided into two parts: short inter-ring movements for transition between TSP tours in adjacent planning periods followed by long intra-ring movements for time allocated for robots to patrol the TSP tour.

1) Inter-Ring and Intra-Ring Movements: Let us begin with inter-ring movements. Before the planning period starts, each robot pair computes the TSP tour. All robot pairs actually share the same TSP tour from the synchronized D-SPOG. Inter-ring movements allow robots to move from current TSP tour to the next. Each robot has a pre-allocated beginning position on the TSP tour (detailed later as initial positions for intra-ring movements). Therefore, the amount of travel time for inter-ring movement can be predicted as soon as the TSP tour for the planning period is established. Define $D_{u,\kappa}$ as the inter-ring travel time of the u-th robot pair. To synchronize the starting time of intra-ring movements of all robot pairs, every robot pair waits until all other robot pairs reach at their initial positions. Define D_κ^{max} as the maximum travel time: $D_\kappa^{max} = \arg\max_u D_{u,\kappa}$. For those robots that arrive early due to short traveling distance, they need to wait $\omega_u^\kappa = D_\kappa^{max} - D_{u,\kappa}$ before the synchronized intra-ring movements start. Synchronization will be detailed later in Section V-B.3. To save time for patrolling TSP tours, robots moves at their fastest speed to shorten inter-ring movement time.

Now let us introduce intra-ring movements. Since the TSP tour is a continuous loop, it can be mapped to a circular ring in time with its circumference being the time for a single pair of robots to traverse the entire TSP tour, which is defined as τ_0. The mapping is one-to-one if we fix a point correspondence in the mapping. For example, the leftmost point (the smallest in lexicographic order) on the TSP tour corresponds to the 9 clock position on the time ring as the green stars shown in Figs. 2(a) and 2(b). All robots share this mapping rule to synchronize their positions on the time ring. The introduction of time ring can facilitate our planning. Under the time ring, the inter-ring movements can also be simplified as shown in Fig. 2(d).

As illustrated in Fig. 2(b), each pair of robots are evenly distributed on the time ring. Define $\phi_{u,\kappa}$ and $\phi'_{u,\kappa}$ as the position and speed of the u-th robot pair on the time ring, respectively. Robots' speeds on the time ring are unitary based on the definition of the time ring. Odd and even pairs are initially assigned to move on the time ring counterclockwise and clockwise, which are represented as 1 and -1, respectively. Recall there are m robots and hence $m/2$

pairs. We have

$$\phi_{u,\kappa} = \frac{2\tau_0(u-1)}{m}, \quad \text{and} \quad \phi'_{u,\kappa} = \begin{cases} 1 & \text{if } u \text{ is odd,} \\ -1 & \text{otherwise,} \end{cases} \tag{6}$$

as the initial positions and speeds for robot pairs. Fig. 2(b) illustrates initial positions and directions (represented by the heading direction of each robot) of four robot pairs. When two robot pairs rendezvous on the time ring, they exchange information and then reverse their moving directions. Therefore, each robot pair oscillates on the time ring centered at its initial position as shown in Fig. 2(c).

Define T as the time of the intra-ring movements. The robot pairs have to execute the intra-ring movements long enough to ensure the existence of the checkpoint.

Lemma 2: Each robot pair has at least one checkpoint if the intra-ring movement time T is

$$T = \begin{cases} \frac{\tau_0}{2} & \text{if } \frac{m}{2} \text{ is even,} \\ \frac{\tau_0}{2} + \frac{\tau_0}{m} & \text{otherwise.} \end{cases} \tag{7}$$

Proof: Starting from its initial position, robot pair u meets its two neighbors $u-1$ and $u+1$ and returns to the initial position with a period of $\frac{2\tau_0}{m}$. The furthest point is half circle away, $\tau_0/2$. The two rendezvous bring information from further both downstream (from $u+1$ at lower half circle) and upstream (from $u-1$ at upper half circle). Imagine the information is sent out from the furthest robot pair from both upstream and downstream directions. When the information reaches the robot pair u, it contains information from all robot pairs. There are two cases: even and odd numbers of robot pairs. For the even case, due to the unitary speed, $\tau_0/2$ is the exact time when robot pair u gathers the information. Eq. (5) has a nonnegative solution and a new checkpoint is established. For the odd case, the proof is similar except that there needs to be an additional half period for meeting the additional pair. ∎

Fig. 1 illustrates the information flow and checkpoint existent for the four robot pair case in Fig. 2(b) and Fig. 2(c) under the oscillating intra-ring movements.

2) Memory Usage and Expected Searching Time: DPRWA ensures periodical checkpoint existence which leads to guaranteed performance. To measure the algorithm performance, we employ two metrics: memory usage for each robot and the expected searching time for each radio source.

For the memory usage, following Lemma 2, we have the following theorem,

Theorem 1: DPRWA guarantees D-SPOG has a time delay less than $D_\kappa^{max} + T$ if comparing the D-SPOG to the centralized SPOG. To achieve that, each robot requires $O(n + m(D_\kappa^{max} + T))$ memory space.

The expected searching time for a radio source has to depend on the source transmission rate. Assume a radio source i transmits signals according to a Poisson process with a rate of λ_i. In [24], we have introduced the expected searching time (EST) for a single-robot-single-target case. Let us extend this analysis to DPRWA. Denote T_s as the searching time. Similar to the EST analysis of RWA in [22], we tighten the convergence condition from the probability
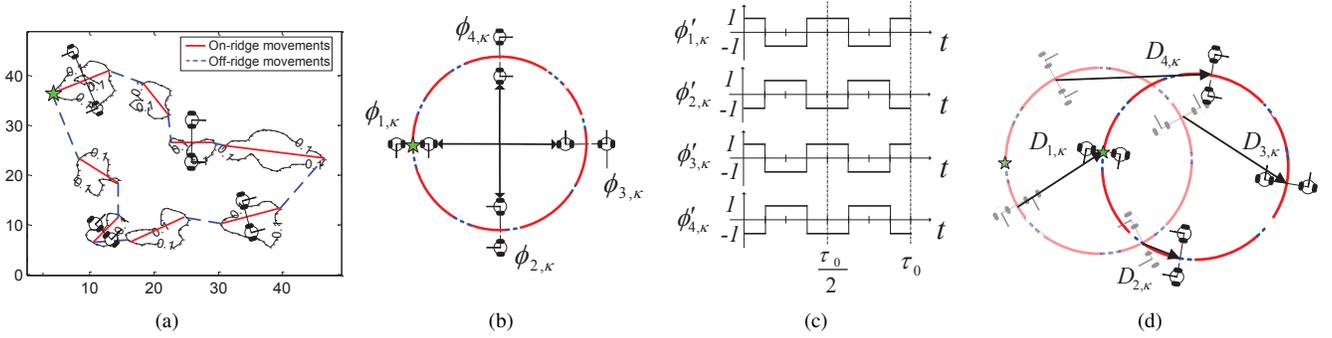
Fig. 2. Sample results for the decentralized planning using 4 robot pairs. (a) Sample robot trajectories of DPRWA. The solid red and dashed blue lines represent the on-ridge and off-ridge movements, respectively. (b) An example of the intra-ring movements on the time ring. (c) Changes of robot pair directions corresponding to the intra-ring movements. (d) An example of the inter-ring movements using time ring space instead of Euclidean space.

threshold p_t to the condition of signal saturation. Radio source i is considered to be found if the robot pairs hear the transmission within the distance of d_a of the radio source. d_a is set to be small such that if the transmission is heard, the probability threshold p_t must be reached. This defines *a sensing circle* with its center at the radio source i and a radius of d_a. Define τ_{IN} and τ_{OUT} as portions of the time when traveling within and outside distance of d_a of radio source i, respectively. Hence

$$D_\kappa^{max} + T = \tau_{\text{IN}} + \tau_{\text{OUT}}. \qquad (8)$$

We have the following theorem,

Theorem 2: The expected searching time $E(T_s)$ of radio source i has the following upper bound,

$$E(T_s) \leq D_\kappa^{max} + \frac{\tau_0}{m} + \frac{1}{\lambda_i} + (D_\kappa^{max} + T)E\Big(\frac{e^{-\lambda_i \tau_{\text{IN}}}}{1 - e^{-\lambda_i \tau_{\text{IN}}}}\Big). \qquad (9)$$

Proof: From Theorem 1 in [24], the expected searching time $E(T_s)$ of transient radio source i is,

$$E(T_s) = E(D) + \frac{1}{\lambda_i} + E\Big(\tau_{\text{OUT}}\frac{e^{-\lambda_i \tau_{\text{IN}}}}{1 - e^{-\lambda_i \tau_{\text{IN}}}}\Big), \qquad (10)$$

where D is the amount of time from the beginning of the search to the moment that the robot is within distance d_a of the radio source i for the first time. The theorem is built on the general case that the searching process can be modeled as a delayed alternative renewal reward process [26]. In our case, the renewal period starts at the first entry to the sensing circle by the robot team at each planning period. It is not exactly the same as the planning period but will share the same expected period length with the planning period. Since the search begins with the inter-ring movements in the first planning period, let us define probability event B as the event if the robot team meets the radio source during inter-ring movements. Therefore,

$$E(D) = E(D|B)P(B) + E(D|\overline{B})(1 - P(B)). \qquad (11)$$

Event B is a small probability event given the large searching field size. Hence $P(B) \ll 1 - P(B) \approx 1$. Since the point of the first entry to the sensing circle could be anywhere on the time ring, it is uniformly distributed on the time ring. Also,

we have $m/2$ pairs of robots evenly distributed on the time ring,

$$E(D|B) = D_\kappa^{max} + \frac{1}{2}\frac{\tau_0}{m/2} = D_\kappa^{max} + \frac{\tau_0}{m} \geq E(D). \qquad (12)$$

Since the search region is usually much larger than the sensing region $\tau_{\text{IN}} \ll D_\kappa^{max} + T$, we have

$$E(\tau_{\text{OUT}}) \approx D_\kappa^{max} + T \qquad (13)$$

from (8). Since each renewal period is independent and identically distributed (*i.i.d.*), it allows us to apply Theorem 1 in [24]. Since $D_\kappa^{max} + T$ is independent of τ_{IN}, plugging (12 and 13) into (10), we have the result in (9). ∎

Remark 1: An important result given by Theorem 2 is the fact that entries in (9) are not sensitive to the number of radio sources. This means that our DPRWA algorithm has excellent scalability when number of radio sources increases.

3) Algorithm: We summarize our DPRWA as follows. Note that this algorithm runs on each robot pair, which skips the details of the internal coordination within each pair.

Synchronization: The algorithm runs at t_κ, the beginning of planning period κ. The algorithm relies on the D-SPOG at $t_{\kappa-1}$, which is the synchronized belief function across all robots. Therefore, all robots will have the same TSP tour, which ensures their motions are synchronized given the same plan, accurate clocks from GPS, and the same mapping rule between the time ring and the Euclidean space.

Virtual ridges: One point that we have yet to explain is the virtual ridge mentioned in line 2 of Alg. 1. Define $s_{\max}$ as the maximum number of ridges. If there are not enough ridges generated from the D-SPOG, we employ virtual ridges to ensure that there are $s_{\max}$ ridges. The main reason is that we do not want the robot to only explore regions with high probabilities. Virtual ridges are generated uniformly random in the searching region and also refreshed at every planning period. The virtual ridge sets are synchronized in the same way that D-SPOG does. The introduction of virtual ridge can be simply viewed as a sampling approach to cover regions with low probabilities.

Sparse sensor fields: The DPWRA in Alg. 1 forces all robots to share a single TSP tour. This is efficient when
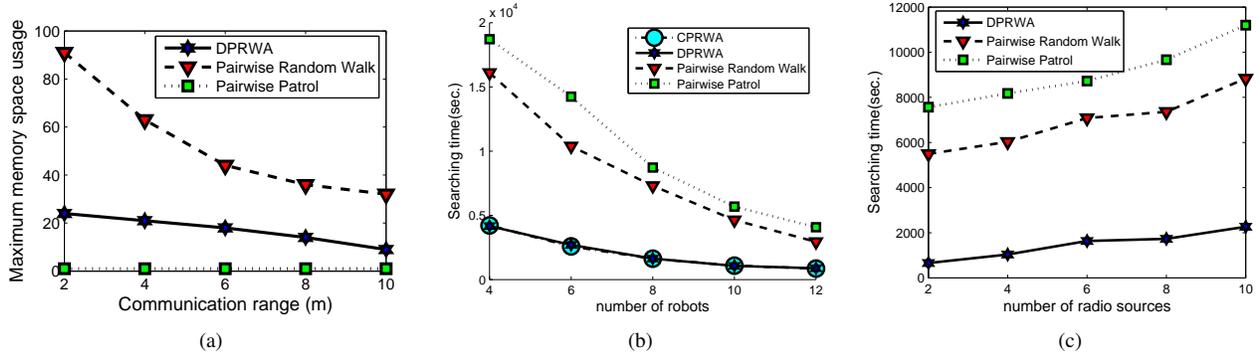
6

Fig. 3. Experiment results when comparing the DPRWA, the pairwise random walk, and the pairwise fixed-route patrol: (a) The maximum memory space usage using number of detection events stored on-board. (b) and (c) Searching time comparison while changing number of robots (b) or number of radio sources (c).

Algorithm 1: Decentralized Pairwise Ridge Walking Algorithm

	input : D-SPOG at $t_{\kappa-1}$	
1	Apply the level set	$O(n)$
2	Compute ridges and merge them with pre-generated virtual ridges	$O(s_{\max})$
3	Compute the TSP tour from the merged ridge set	$O((s_{\max}-1)!)$
	// Inter Ring Movements	
4	Compute $D_{u,\kappa}$ and D_{κ}^{max}	$O(1)$
5	Move to initial positions	$O(1)$
6	Wait ω_u^κ	$O(D_\kappa^{max})$
	// Intra Ring Movements	
7	**while** $t \le t_\kappa + D_\kappa^{max} + T$ **do**	$O(T)$
8	$\quad$ Patrolling along the TSP tour	$O(1)$
9	$\quad$ **if** $\phi_{u,\kappa}^{j,k} = \phi_{u-1,\kappa}^{j,k}$ or $\phi_{u,\kappa}^{j,k} = \phi_{u+1,\kappa}^{j,k}$ **then**	
10	$\quad\quad$ $\phi_{u,\kappa}^{'j,k} = -\phi_{u,\kappa}^{j,k}$	$O(1)$
11	$\quad\quad$ Update $\Gamma_{u,\kappa}^{j,k}$	$O(1)$
12	$\quad$ **end**	
13	**end**	

radio sources are relatively dense (i.e. the distances between disconnected components in the level set are less than the communication range). However, sharing a TSP tour might not be efficient when the radio source are sparsely distributed in the searching region because robots have to waste a lot of time on off-ridge movements running between radio sources. Since the communication range is much larger than the sensing range given that robots have more power and better antenna than radio sources, the sensing processes in D-SPOG are independent across distant groups, which allows us to partition D-SPOG spatially into disjoint distant groups. Each group is treated as a separated problem with no requirement to merge D-SPOG during the partition. We can regroup periodically should inter-group distances change. At the moment of re-grouping, we can merge D-SPOG across groups. Robot pairs will be proportionally dispatched to different groups according to the total $P(C_i|\mathcal{Z}^{1:k})$ of each group. We may have to merge some close groups when there is an insufficient number of robot pairs. For each group, we apply Alg. 1.

VI. EXPERIMENTS

To validate the algorithm, we have implemented the algorithm and a simulation platform. The radio sources are XBee Pro with ZigBeeT/802.15.4 OEM radio frequency modules produced by Digi International Inc. We use the RSS readings from XBee Pro to drive the simulation experiments. We cannot use real robots because we do not have robots that can patrol the searching field of about a football field size for more than 20 hours, which is a basic requirement for a realistic setup. We simulate iRobot Create in the process, which has a maximum speed of 40 cm/s. The grid is a square with 50×50 cells. Each grid cell has a size of 50.0×50.0 cm^2. Each radio source generates radio transmission signals according to an *i.i.d.* Poisson process with a rate of $\lambda = 0.05$ packets per second. The radio sources also dynamically vary their transmission power using one of 5 power settings in XBee Pro, which results in a varying sensing range from 1.67 to 3.45 meters. We set $\tau_0 = 500$ seconds in the simulation. We choose the probability convergence threshold as $p_t = 0.9$. During each trial, we randomly generate radio source locations in the grid.

We compare the DPRWA algorithm to two heuristics including a pairwise random walk and a pairwise patrol. In both heuristics, robots are paired just as DPRWA does. In the former, each pair is treated as a super robot to perform a random walk together. In the later, robot pairs follow a linear formation with an equal inter-pair distance to be the maximum communication distance. Since global connectivity is maintained for the pairwise patrol, it degenerates to the centralized planning.

Fig. 3 illustrates the simulation results by using memory usage and searching time as metrics while changing communication range, number of robots, and number of radio sources. Each data point is an average of 20 independent trials. There are six radio sources to be searched in Figs. 3(a) and 3(b). In Figs. 3(a) and 3(c), eight robots are employed. The communication range is set to be six meters in Figs. 3(b) and 3(c). Since the pairwise patrol maintains global connectivity, it requires the least amount memory for synchronization purpose. The pairwise random walk is the

opposite because the time between checkpoints for robots can be very long. Our DPWRA requires more memory than that of the patrol but still much less than that of the random walk (see Fig. 3(a)). When it comes to the searching time, DPRWA is significantly faster than its counterparts (see Figs. 3(b) and 3(c)). The advantage is even more when number of robots are limited, which happens when the search is constrained by resources. Fig. 3(b) also compares DPRWA with the centralized PRWA (CPRWA) in [25]. It is surprising that DPRWA EST is about the same as CPRWA despite the advantage that CPRWA has in coordination and synchronization. Fig. 3(c) further confirms Theorem 2 that the EST of DPRWA is insensitive to number of radio sources. In conclusion, DPRWA successfully trades a modest amount of memory for the fastest searching time among the three methods along with excellent scalability due to its invariance to number of radio sources.

VII. Conclusion

We developed a decentralized algorithm to coordinate a group of mobile robots to search for unknown and transient radio sources in an open field under mobility, communication range, and sensing constraints. Building on our prior work, we proposed a two-step approach: first we decentralized belief functions that robots use to track source locations using checkpoint-based synchronization, and second we proposed a decentralized planning strategy to coordinate robots to ensure the existence of checkpoints and coordinated searching. We formally analyzed memory usage, data amount in communication, and searching time for the proposed algorithm. We implemented the proposed algorithm and compared it with two heuristics in simulation based on real sensory data. Our algorithm successfully performed the searching task and showed a clear advantage in searching time without significant increase of memory usage.

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